Biyani Girl's College

Concept Based Notes

DISCRETE MATHEMATICS

BSC Semester I

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BIYANI GIRLS COLLEGE

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Preface

I am glad to present this book, especially designed to serve the needs of the students. The book has been written keeping in mind the general weakness in understanding the fundamental concepts of the topics. The book is self-explanatory and adopts the “Teach Yourself” style. It is based on question-answer pattern. The language of book is quite easy and understandable based on scientific approach.

Any further improvement in the contents of the book by making corrections, omission and inclusion is keen to be achieved based on suggestions from the readers for which the author shall be obliged.

I acknowledge special thanks to Mr. Rajeev Biyani,Chairman & Dr. Sanjay Biyani, Director (Acad.) Biyani Group of Colleges, who are the backbones and main concept provider and also have been constant source of motivation throughout this endeavor. They played an active role in coordinating the various stages of this endeavor and spearheaded the publishing work.

I look forward to receiving valuable suggestions from professors of various educational institutions, other faculty members and students for improvement of the quality of the book. The reader may feel free to send in their comments and suggestions to the under mentioned address.

Author

Syllabus

S.No.	Unit	Topic
1	1.1	Relation on set
	1.2	Equivalence class
	1.3	Partial order relations
	1.4	Chains and Anti-chains
	1.5	Lattices, Distributive and Complemented Lattices
	1.6	Boolean algebra
	1.7	Conjunctive normal form, Disjunctive normal form
	1.8	Pigeon hole principle
	1.9	Principle of inclusion and exclusion
	2.0	Propositional calculus
	2.1	Basic logical operations
	2.2	Truth tables
	2.3	Tautologies and contradictions
2	2.4	Discrete numeric functions
	2.5	Generating functions, Recurrence relations
	2.6	Linear recurrence relation with constant coefficients and their solutions
	2.7	Total solution
	2.8	Solution by method of generating functions
	2.9	Basic concepts of graph theory
	3.0	Types of Graph, Planar Graph
	3.1	Walks, Paths and Circuits
	3.2	Shortest Path problem
3	3.3	Planar graph, Operation on Graph
	3.4	Matrix representation of graph
	3.5	Adjacency matrices, Incidence matrices
	3.6	Hamiltonian and Eulerian Graph
	3.7	Tree, Spanning Tree
	3.8	Minimum Spanning Tree
	3.9	Distance between vertices
	4.0	Centre of tree Binary tree Rooted tree
	4.1	Binary tree
	4.2	Rooted tree

4	4.3	Linear Programming Problems
	4.4	Basic solutions
	4.5	Some basic properties and theorems on convex sets
	4.6	Simplex algorithm
	4.7	Two-Phase method
	4.8	Duality
	4.9	Solution of dual problems
	5.0	Transportation problems
	5.1	Assignment Problems

Short Questions:

Question 1: Ifaset A={1,2}.Determineallrelations fromAtoA.

Solution: Thereare2^2=4elementsi.e.,{(1,2),(2,1),(1,1),(2,2)}inA×A.So,therea re2^4= 16relationsfromAtoA.i.e.
{(1, 2), (2, 1),(1,1),(2,2)},{(1, 2), (2,1)},{(1,2),(1,1)},{(1,2),(2, 2)},
{(2,1), (1, 1)},{(2,1),(2, 2)},{(1, 1),(2,2)},{(1,2), (2, 1),(1, 1)},{(1,2),(1,1)
(2, 2)}, {(2,1),(1,1),(2,2)},{(1, 2), (2,1),(2,2)},{(1,2),(2,1), (1, 1),(2,2)}

Question 2: SupposetherelationR={(1,a),(1,b),(3,b),(3,d),(4,b)}fromX= {1,2,3,4}to Y = {a,b, c,d}.Statethe domainandrange ofrelationR.

Solution: WehaveR={(1,a),(1,b),(3,b),(3,d),(4,b)}
X={1, 2,3,4}
Y={a,b,c, d}

The domain of (R) = {1, 3, 4}
Therelationof(R) ={a, b,d}

Question 3: IfR={x– 2,2x+3}andx∈{0,1,2,3,4,5}thanfindthedomainandrangeofthe relationR.

Solution: We have x = {0, 1, 2, 3, 4, 5}R= {x –2, 2x+ 3}
x=0⇒x– 2=0–2 =–2and 2x+ 3 = 2×0 + 3=3
x=1⇒x– 2= 1– 2 =–1 and2x+3 = 2×1+3=5
x=2⇒x– 2= 2– 2 =0 and 2x+3 = 2×2+3 =7
x=3⇒x– 2= 3– 2=1 ad2x+3 = 2×3+3 = 9
x=4⇒x– 2= 4– 2 =2 and 2x+3=2×4+3 =11
x=5⇒x– 2= 5– 2 =3 and 2x+3 =2×5+3 =13

ThereforeR={–2,3),(–1,5),(0,7),(1,9),(2,11),(3,13)
DomainofR={–2,–1, 0,1,2, 3}
Rangeof R= {3,5,7,9,11,13}

Question 4: Explain Equivalence Relation.

Solution: If the relation R is reflexive, symmetric and transitive then this relation is referred as**equivalencerelation.** The equivalence relationisa relationshipon theset andit isdenoted by"∼".

1. **Reflexive:** Foreverya ∈A,if(a,a)∈Rthenthisrelationisreflexive.

   Proof: Forallpairsofpositiveintegers,((a, b),(a,b))∈R
   Clearly, wecansay ab=abforallpositiveintegers.

   Wecanclearlysaythatforallpositiveintegersab =ab.Henceproved

2. **Symmetric**:If(a, b)∈R,then(b,a)∈Rthenthisrelationis symmetric.

   Proof: From the symmetric property,When(a, b) ∈R,thenwehave(b,a)∈R
   Forthegivencondition,
   If((a, b),(c,d))∈R,then((c, d),(a,b))∈R.If((a, b),(c,d))∈R,thenad =bcand cb =da

   AsmultiplicationiscommutativeHence((c,d),(a,b))∈R

   Henceproved

3. **Transitive**:If(a, b)∈Rand (b,c)∈R,then(a, c)∈Rthen thisrelationistransitive.

   Proof: For the given set of ordered pairs of positive integers,((a, b), (c,d))∈Rand((c,d),(e, f))∈R,
   Then((a,b),(e, f)∈R.

   Suppose((a, b),(c,d))∈Rand((c, d),(e, f))∈R.Then wehave,ad = cb andcf = de.

   Fromaboverelationa/b =c/dand thatc/d =e/fHenceifa/b = e/fthen we obtainaf= be.

   Hence((a,b),(e,f)) ∈R.Thereforethispropertyisproved.

Question 5: Explain Partial Order Relation.
Solution: Ifanyrelationthatsatifiesthebelowthreeproperties,

1) RelationRisReflexive,i.e.aRa∀a∈A.
2) RelationRisAntisymmetric,i.e.,aRband bRa⟹a=b.
3) RelationRistransitive,i.e.,aRbandbRc⟹aRc.

ThenthisrelationRonaset Aistermed asa**partialorderrelation**.

Question 6: What do you mean by Hasse Diagram? And write ProcedureforDrawingHasseDiagram.

Solution:
Adirectedgraphinwhichtheorderedbetweentheelementsarepreservedisknown asHassediagram.Ontheotherhand,Aposet(X,≤)isrepresentedbyadiagram called

Hasse Diagram.
ProcedureforDrawingHasseDiagram

ThefollowingstepsdescribesforthedrawingHassediagram:

Step1) Drawthedigraphofgivenrelation.
Step2)*Deleteallcyclesfor digraph.*

Step3)Eliminatealledgesthatareimpliedbythetransitiverelation.

Step4) Draw the digraph of a partial order with edges pointing upward so that arrowsmay be omitted from edges.
Step5) Replacethecirclesrepresentingtheverticesbydots.

Question 7: Find all chains and antichains of the given poset{(a, b), (a, c), (a, d), (b, c), (b, d) (a, a), (b, b), (c, c), (d, d)}.

Solution: Suppose the poset is {(a, b), (a, c), (a, d), (b, c), (b, d) (a, a), (b, b),(c, c), (d,d)}.

Thechainsintheposetwithmorethanoneelementare:

{a,b,c}{a,b,d}
{a,b), {b,c},{b,d}

Theantichainsintheposetwithmorethanoneelementare:{c,d}.

Question 8: Showthateverychainisdistributivelattice.

Solution: SupposeLisa chainandsupposea,b,c $\in$L. SinceLisa chain,anytwoofa,b and c are comparable. Assume the following possible cases:

1) a$\leq$ bora $\leq$cand
2) a$\geq$ band a$\geq$c

Forcase(1),i.e.,a$\leq$bora $\leq$c

a$\wedge$(b$\vee$c)=aand(a $\wedge$b)$\vee$(a$\wedge$c)=a$\vee$a=a

Hence $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

For case (2), i.e., $a \geq b$ and $a \geq c$ $a \wedge (b \vee c) = b \vee c$ and $(a\wedge b)\vee(a\wedge c)=b\vee c$

Hence$a\wedge(b\vee c)=(a\wedge b)\vee(a\wedge c)$.

Question 9: Prove that L be a lattice with element 0 and greatest element 1. Then show that 0 is the unique complement of 1 and 1 is the unique complement of 0.

Proof: Aswe knowthat0 $\wedge$1=0and0 $\vee$1=1

Hence, 0 and 1 are complements of each other. Now we prove that 0 and 1 have the unique complements. First, we shall showthat the complement of0 unique. Suppose that a is any other complement of 0. Then 0 $\wedge$ a = 0 and 0 $\vee$ a = 1.
Butwe knowthat0$\vee$a=a$\Rightarrow$a=1

Thus,thecomplementof0isunique.Similarly,wecanprovethatthecomplement of 1 is unique.

Question 10: Findoutallthesub-latticesofD$_{30}$thatcontainatleastfourelements,

D_{30} = {1, 2, 3, 5, 6, 10, 15, 30}.

Solution: Thesub-latticesofD$_{30}$thatcontainatleastfourelementsareasfollows:

1) {1,2,6,30} 2) {1,2,3,30} 3) {1,5,15,30} 4){1,3,6,30}

5) {1,5,10,30} 6) {1,3,15,30} 7) {2,6,10,30}

Question 11: Define Objective of the problem.

Solution: To maximize the profit how much of X and Y are to be manufactured?That is **maximization of the profit or maximization of the returns** is the objective of the problem. For this in the statement it is given that the profit contribution of X is Rs 5/- per unit and that of product Y is Rs. 7/- per unit.

Long Questions:

Question1. SupposeA={2,3,4,5}andB={8,9,10,11}. Supposetherelation„isfactorof"fromAtoBisR.

1) **WriteRintherosterform. Also,determinetheDomainandRangeofR.**
2) **Drawanarrowdiagramtodepictthe givenrelation.**

Solution:

1) SupposeRhaselements(a, b)whereaisafactorofb.

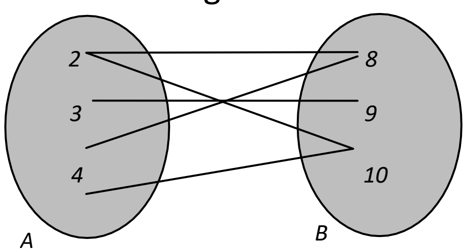
   Therefore,Relation(R)intherosterformisR={(2,8);(2,10);(3,9);(4,8),(5,10)}

   HencetheRelation(R)inroster formis givenasR={(2, 8);(2, 10);(3,9);(4,8), (5,10)}

   Therefore,Domain(R)=SetofallfirstcomponentsofR={2,3,4,5}andRange (R)=Setofallsecond componentsofR={8, 10,9}

   Hence, thedomain(R)is theset ofall firstcomponents ofRandtheRange(R)isthesetofall secondcomponents.
   Domain(R) ={2, 3,4, 5}
   Range(R)={8,10,9}

2) Thearrowdiagramthat showsRisasfollows*:*



Question2. SupposeA isasetofnon-zerointegersandassumethat≈istherelationonA×A whichisgivenas
(a, b)≈(c,d) whenever

ad

=bcVerifythat≈isanequivalencerelation.

Solution: Wehavetoverifythatis≈isequivalent,symmetricandtransitive.

i) ***Reflexivity:*** *Given that (a, b) ≈ (a, b)Asab= ba*

   ∴≈isreflexive.

ii) ***Symmetry:*** *We have (a, b) ≈ (a, b)Sincead=bc andcb = da*

   Thus,(c, d)=(a,b).

   ∴≈issymmetric.

iii) ***Transitivity:*** *Let (a, b) ≈(c, d) and (c, d) ≈(e, f).Thenad= bc andcf= de.*

Onmultiplyingcorrespondingtermsoftheequationswe get(ad)(cf)= (bc)(de)

Terminatec ≠0 and d≠ 0onbothsidesofthe equationwegetaf= be,andhence(a,b)≈(e,f).

∴≈istransitive.

Therefore≈isanequivalencerelation.

Question3. : OnthesetofintegersI, therelationaRb isan equivalence relationifa-bisamultiple of5.Determine theequivalenceclasses.

i) **Solution:** *TherelationaRbifa–bisamultipleof5andthisrelationisreflexivebecausefora* ∈I,a – a=0,multipleof5.Hence,aRa.

ii) *The relation aRb is symmetric because for a, b ∈ Ia –bisa multiple of5,* andb–aisalsoamultipleof5Hence,

∴aRb⇒bRa,∀a,b∈ I.

iii) *TherelationaRbiffa–bisamultipleof5,istransitivebecausefora,b,c ∈Iifa–b isamultipleof5, thenb–c isalso amultipleof5,*

Thus,

∴ (a –b)+(b– c)isamultipleof5,

⇒ a– cisamultipleof5.

∴aRband bRc⇒aRc.

The relation aRb if a – b is a multiple of 5 it impliesthat a ~ b.Ifa–b= 5k,Where kisaninteger

Ifwedividetheinteger aby5thentheremainder isb.Itisclear thatthe remainder canbe0, 1,2, 3or4.

Theequivalenceclass;

[a]={x:x∈I,xRa}

={x:x∈I,x– a=5k}

[0]={x:x ∈ I,x– 0=5k}

={x:x∈I,x=5k},wherek =0,±1, ±2,…..

={0, ±5, ±10, ±15,….}

[1]={x:x ∈ I,x– 1=5k}

={x:x∈I,x=1+5k}

={1, 1±5,1 ±10,1±15, ….}

= {….,–9,–4,1, 6,11,16,….}

[2]={x: x∈I,x– 2 = 5k}

$=\{2, 2 \pm 5, 2\pm10, 2\pm15, \ldots.\}$
$=\{\ldots., -8,-3,2,7,12,17,\ldots.\}$[3]=
$\{x:x \in I, x-3=5k\}$
$=\{3, 3 \pm 5, 3\pm10,\ldots.\}$
$=\{\ldots., -7,-2,2,3,8,13,\ldots.\}$[4]=
$\{x:x \in I, x-4=5k\}$
$=\{4, 4\pm5, 4\pm10, \ldots.\}= \{-11,-6, -1, 4,9,14,\ldots.\}$

Accordingtoequivalenceclasses wehave[0],[1],[2],[3],[4]

Question4.
IfRistherelationonthesetofintegerssuchthat(a,b)$\in$R,ifandonlyif3a+ 4b =7nfor someintegern,prove thatRisanequivalencerelation.
Solution: Wehave3a+4a=7a, hereaisaninteger.
$\therefore (a,a) \in R$

ThereforetherelationRisreflexive.
3b + 4a =7a +7b–(3a +4b)
=7(a+b)–7n= 7(a+b– n),wherea +b– nisaninteger.
Thus,(b,a)$\in$Rwhen(a,
b)$\in$R.ThereforetherelationRissymmetr
ic.Suppose(a, b) and (b,c)$\in$R.
Thenconsider 3a+4b=7m .….(1)
and 3b+ 4c=7n .….(2)

Fromequation(1)andequation(2)wehave,
3a+4c=7(m+ n – b),where(m+n–b)isaninteger.
Thus,(a,c)$\in$R
ThereforetherelationRistransitive.Hencerel
ationRisanequivalencerelation.

Question5. Ifwehaveorderedpair(4,6),(8,4),(4,4),(9,11),(6,3),(3,0),(2,3),find thefollowingrelations.Also, determinethe domainandrange:

1)Istwo lessthan 2)Islessthan

3)Isgreater than 4)Isequal to

Solution:

1) R_1thesetofallorderedpairsisR_1whose1st componentistwolessthanthe2nd component.

Hence,R_1={(4,6),(9,11)}

Also, Domain (R_1) is the Set of all first components hence Domain (R_1) = {4, 9}Range(R_2) istheSetofallsecond componentshence R_2={6,11}

2) TheSetofallorderedpairsisR_2whose1st componentislessthanthesecondcomponent.
Hence,R_2={(4,6),(9,11),(2,3)}.
Also,Domain(R_2) ={4,9,2}
Range(R_2)={6, 11,3}

3) TheSetofallorderedpairsisR_3whose1st componentisgreaterthanthesecondcomponent.

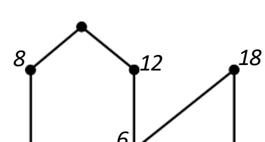
Hence,R_3={(8,4),(6,3),(3,0)}

Also,Domain(R_3) ={8,6,3}
Range(R_3)={4,3,0}

4) *Thesetofallorderedpairs isR_4whose1st componentisequaltothesecondcomponent.*
Hence,R_4=
{(3,3)}Also,Domain(R)={3}
Range(R) ={3}

Question5: Let A= {1, 2, 3, 4, 6,8, 9, 12, 18, 24}be ordered bythe relation “a divides b”. Draw the Hasse diagram of (A, ≤), where a ≤ b ⇔a|b i.e., a divides b.

Solution: TheHassediagramisgivenbelow,

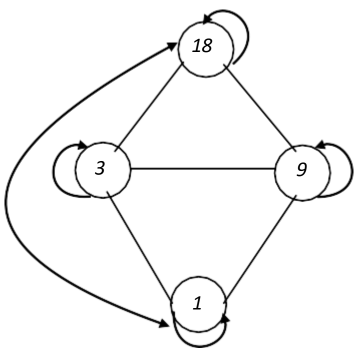


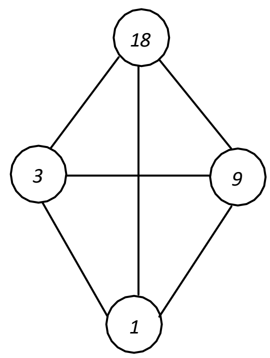
Question6: If the set is A = {1, 3, 9, 18} and B = {3, 5, 30} then draw the Hasse diagramofthegivensetsunderpartialorderingrelation„dividesandindicateth osewhich are chains

Solution: WeknowthatA={1,3,9,18}. The relation is given by,

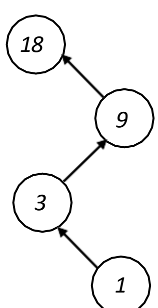
R={(1,1),(1,3),(1,9),(1,18),(3,3),(3,9),(3,18),(9,9),(9,18),(18,18)}

TheDigraphis,





RemoveTransitiveedges



TheHouseDiagramis,

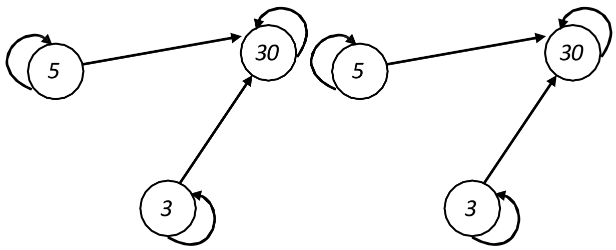
Inthisposeteachtwoelementsarerelated thereforeitisachain.

Therefore, this is totally ordered as well linearly ordered set.
ForB={3,5,30}

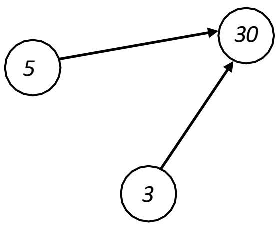
TheRelationis givenas,R={(3,3),(3,30),(5,5),(5,30),(30,30)

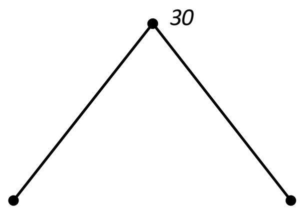
Now Remove Loops,

TheDigraphis



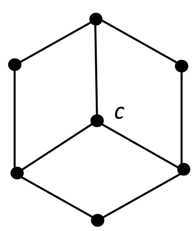
RemoveTransitiveedges The Hasse Diagram is





Herewehaveobservethat{5,30}and{3,30}arechain{5,3}isanti-chain.

Question6.
FindoutwhetherthefollowingHassediagramrepresentlatticeornot.



Solution:

LUB:

∨	a	b	c	d	e	f	g
a	a	a	a	a	a	a	a
b	a	b	a	a	b	a	b
c	a	a	c	a	c	c	c
d	a	a	a	d	a	d	d
e	a	b	c	a	e	c	e
f	a	a	c	d	c	f	f
g	a	b	c	d	e	f	g

GLB:

∧	a	b	c	d	e	f	g
a	a	b	c	d	e	f	g
b	b	b	e	g	e	g	g
c	c	e	c	f	e	f	g
d	d	g	f	d	g	f	g
e	e	e	e	g	e	g	g
f	f	g	f	f	g	f	g
g	g	g	g	g	g	g	g

Eachsubsetoftwoelementshasaleastupperboundandagreatestlowerbound,soitis a lattice.

Theorem7:Let(L,≤)bealattice.ThenLsatisfiesthefollowinglaws:

1) ***IdempotentLaws:*** *a∧a=a.and,a ∨a=a, ∀a ∈L.*
2) ***CommutativeLaws:*** *a∧b=b∧aand,a∨b=b ∨a, ∀α,b ∈L.*
3) **AssociativeLaws:**
   (a∧b)∧c=a ∧(b∧c)and,(a∨b)∨c=a∨(b ∨c).∀a, b,c ∈L.
4) ***AbsorptionLaws:*** *a∧(a∨b)=a,and,a ∨(a∧b)=a. ∀a.b ∈L.*

Proof:

1) *Supposethata ∈L.thedefinitionofg.l.b.isx ∧y≤xforallx,y, z ∈L. And if z ≤ x, and z ≤ y, z ≤ x ∧ y and take x = y = z = a.*

   Thenwe get:a∧α≤aanda ≤α∧a

   And a = a ∧a from anti-symmetry. Similarly, a = a ∨a. Hence the idempotent laws proved.
2) *Supposethata,b ∈L.Thena∧bandb∧aaretheg.l.b.ofaandb.*
   Thesemustbeequal,i.e.,a ∧b=b∧abyuniquenesspropertyoftheg.l.b.
   Similarly, a ∨ b = b ∨ a.
   Hencethecommutativelawsproved.
3) *Suppose that a, b, c ∈ L hence from the definition of g.l.b. (a ∧ b) ∧ c ≤ (a ∧ b), and (a ∧ b) ∧ c ≤c.*

   Consider(a∧b)∧c≤(a∧b)≤aandalsowehave(a ∧b)∧c≤c. Therefore (a ∧ b) ∧ c is the lower bound of a, b and c.

   i.e., (a∧b) ∧c≤a
   (a∧b) ∧c≤(b ∧c)
   ∴(a ∧b)∧c ≤a ∧(b∧c).
   Now, a∧(b ∧c)≤a.
   a∧(b∧c)≤(b∧c)≤b. a ∧ (b ∧ c)
   ≤ (b ∧ c)≤ c
   ∴a∧(b∧c)≤(a∧b)
   ∴a∧(b∧c)≤(a∧b)∧c

   Thereforewehavebytheanti-symmetry,(a∧b)∧c=a∧(b∧c).∀α,b,c∈L.

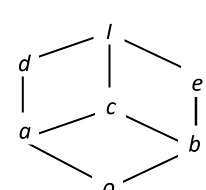
   Similarly we can prove that (a ∨ b) ∨ c = a ∨ (b ∨ c). ∀a, b, c ∈ L.
   Henceassociativelawsproved.
4) *Supposethata,b ∈L.Thena≤aanda≤a∨b.*
   Soa≤a∧(a∨b).Ontheotherhanda∧(a∨b)≤a.

   Therefore, a = a ∧ (a ∨ b).

   Similarly, a=a∨(a∧b).

   Hence absorption laws proved.

Question8. **ShowsthatalatticeLin:**



i) **Whichnon-zeroelementsarejoinirreducible?**

ii) **Whichelementsareatoms?**

WhichofthefollowingaresublatticesofL:

L_1={0,a,b,I} L_3={a,c,d,I}
L_2={0,a,e,I} L_4={0,c,d,I}

i) **IsLdistributive?**

ii) **Findcomplements,iftheyexist,fortheelementsa,bandc.**

iii) **IsLacomplementedlattice?**

Solution:

i) *The elementsa, b, d, and e are joinirreducible because those non-zero elements with a unique immediate predecessor are join irreducible.*

ii) *The elements a and b are the atoms because those elements which immediately succeed 0 are atoms.*

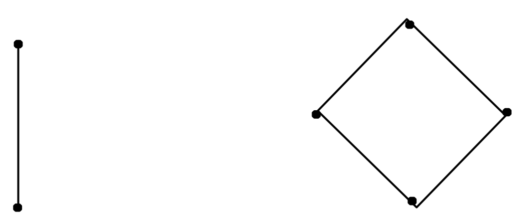
iii) *IfasubsetL'isclosedunder$\wedge$and$\vee$thenitissublattice.SupposeL_1isnotasublatticesincea$\vee$b=c,whichdoesnotbelongtoL_1.Sincec$\wedge$d=adoesnotbelongtoL_4,the set L_4isnotasublattice.Theothertwosets,L_2andL_3,aresublattices.*

iv) *LisnotdistributivesinceM= {0,a,d,e,I}isasublattice.*

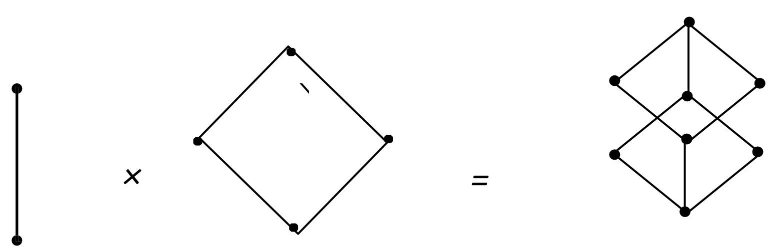
v) *Wehavea$\wedge$e=0anda$\vee$e=I,soaandearecomplements.Alsobanddare complements. However, c has no complement.*

vi) *Lisnotacomplementedlatticesincechas nocomplement.*

Question8. Suppose that L_1 and L_2 are the lattices shown in figures below. DetermineL = $L_1 \times L_2$ and draw the Hasse diagram for L.



Solution:



Question9. Ifacomplement existsthenprovethatitisuniqueinaboundeddistributive lattice.

Solution: Supposethata′anda″isthecomplementsoftheelement.

Ifa $\in$ L

Then, $a \vee a' = 1$ $a \vee a'' = 1$

$a \wedge a' = 0$ $a \wedge a'' = 0$

Nowusingthedistributivelaws,weobtain a′

= a′∨ 0 = a′∨ (a ∧ a″)

=(a′∨a)∧(a′∨a″)

=(a∨a′)∧(a′∨a″)

=1∧(a′∨a″)

=a′∨a″

Also, a″=a″∨0

=a″∨(a ∧a′)

=(a″∨a) ∧(a″∨a′)

=(a∨a″) ∧(a′∨a″)

=1∧(a′∨a″)

=a′∨a″

Hence, a′=a″

Question10.Showthatthelatticeshowninfigureisnotdistributive.

Solution: *If all of lattice elements follow distributive property then lattice is distributive and hence we verify the distributive property between the elements n, l, and m. such as*

GLB(n,LUB(*l*,m))=GLB(n, p) [∴LUB(*l*,m)=p]

=n (L.H.S.)

AlsoLUB(GLB(n,*l*),GLB(n,m))=LUB(o, n);

[∴GLB(n,*l*)=0andGLB(n, m) =n]=n(R.H.S.)

So L.H.S. = R.H.S.

ButGLB(m,LUB(*l*,n))=GLB(m,p) [∴LUB(*l*,n)=p]=m(L.H.S.)

AlsoLUB(GLB(m, *l*),GLB(m,n))=LUB(o,n);[∴GLB(m, *l*)=0andGLB(m,n)=n]

=n (R.H.S.)

Therefore,L.H.S.≠ R.H.S.

Hencedistributivepropertydoesnotholdbythelatticesolattice,isnotdistributive.

Question11. Let (L, ≤) be a lattice with least element 0 and greatest element 1. For any element a∈L, show that,

1) **a∨1=1 anda∧1=a**
2) **a∨0 =aand a ∧0 =0**

Proof:

1)Suppose,„a‟isanyelementofalatticeLandsuppose1isthegreatestelementofLthen a ∨≤ 1 ….(1)

Sincea∨1isthesupremumofaand1,wehave

1≤ a∨1 ….(2)

Fromequation(1)and(2)weget, a ∨ 1 = 1

Again,a∧1istheinfimumofaand1,therefore

a∧1 ≤a ……(3)

Also,sincea≤aanda≤1,then

a≤a∧1 ......(4)

From equation(3)and(4),we get a ∧ 1
= a

3) *Suppose that „a‟ is any element of L and „0‟ be the least element of L then*
∴ 0≤ a and a ≤a
⇒a∨0 ≤a

Although,„a‟ is upper bound of 0 and a while a∨0 is the least upper bound of {0, a}.

Also, from the definition of ∨, we have a
≤a∨ 0

Now, a∨0≤ a and a≤a∨0
⇒a∨0 =a

Again, a∧0 is the infimum of a and 0, therefore a ∧ 0
≤ a

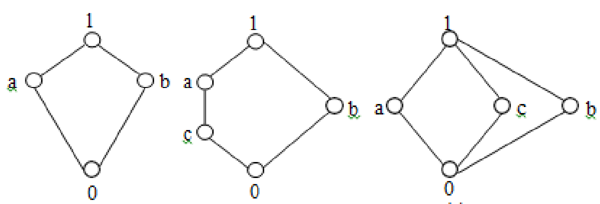
Since 0≤a and 0≤0, we have 0 ≤ a ∧ 0

Now, a∧0≤ 0 and 0 ≤a∧0

⇒a∧0 =0

Question12.

Show that the lattices shown in figure 2.32(a), 2.32(b) and 2.32(c) are complemented lattices.



(a) (b) (c)

Solution:

i) For the lattice (a) GLB(a,b)=0 and LUB(x,y)=1. So, the complement a is b and vice versa. Hence, a complement lattice.

ii) For the lattice (b) GLB(a,b)=0 and GLB(c,b)=0 and LUB(a,b)=1 and LUB(c, b) = 1; so both a and c are complement of b. Hence, a complement lattice.

iii) For the lattice (c) GLB(a,c)=0 and LUB(a,c)=1; GLB(a,b)=0 and LUB(a,b)=1. So, complement of a are b and c. Similarly complement of c are a and b, also a and c are complement of b. Hence lattice is a complement lattice.

Question13. Suppose that (X, ≤) is a distributive lattice now for any

elements x, y ∈ X show that if y is complement of x then y is unique.

Solution: Supposethatelementxhascomplementzotherthanx.So,wehave,

LUB(x, y) = 1 and GLB(x, y) = 0;
andLUB(x,z)=1 and GLB(x,z)=0;

Furtherwewrite,z=GLB(z,I)
⇒GLB(z,LUB(x, y))
⇒LUB(GLB(x,x),GLB(z,y)) (Distributiveproperty)
⇒LUB(0,GLB(z,y))
⇒LUB(GLB(x,y),GLB(z, y))
⇒GLB(LUB(x,z), Y) (Distributiveproperty)
⇒GLB(1,y)⇒y

Thereforecomplementofxisunique.

Question14.

Acompanymanufacturestwoproduct*A*and*B*.Thesearemachinedonmachines *X*and*Y*.*A*takesonehouronmachine*X*andonehouronMachine*Y*.Similarlyproduct*B*takes4hours on Machine *X* and 2 hours on Machine *Y*. Machine *X* and *Y* have 8 hours and 4 hours as idle capacity Theplanningmanagerwantstoavailtheidletimetomanufacture*A*and*B*.Theprofitcontributionof *A*isRs.3/–perunitandthatof*B*isRs.9/–perunit.Findtheoptimalproductmix.

Solution:

MaximizeZ=3a+9bs.t.

1a+4b≤8

1a+2b≤4bothaandbare≥0

Simplexformatis:

MaximizeZ=3a+9b+0S₁+0S₂s.t.

$1a+4b+1S_1+0S_2=8$

$1a+2b+0S_1+1S_2=4$*and*

a,b,S_1,S_2 all ≥ 0.

Table:I.$A=0, b=0, S_1=8, S_2=4$ and $Z=$ Rs.0/–

→

Problem	Profit	Cj Capacity	3 a	9 b	0 S_1	0 S_2	Replacement

variable	*Rs.*						*ratio*
S_1	0	8	1	4	1	0	8/4=2
S_2	0	4	1	**2**	0	1	4/2=2
		Netevaluation	3	9	0	0	

Now to select the outgoing variable, we have to take limiting ratio in the replacement ratio column.But both the ratios are same *i.e*. = 2.Hence there exists a tie as an indication of degeneracy in the problem. To solve degeneracy follow the steps mentioned below:

(*i*) Divide the elements of identity column by column from left to right by the corresponding key column element.

Once the ratios are unequal select thelowest ratio and the row containing hatratio is the key row.

Inthisproblem,forthefirstcolumnoftheidentity(i.e.theS_1column)theratiosare:1/4 ,and

0/2. The lowest ratio comes in row of S_2. Hence S_2 is the outgoing variable. In case ratios are equal go to the second column and try.

***Table:II.**a=0,b=2,S1=0,S2=0,Z=Rs.18/–*

→

Problem variable	*Profit Rs.*	*Cj* *Capacity*	*3* *a*	*9* *b*	*0* *S1*	*0* *S2*	*Replacement ratio*
S_1	0	0	1	–2	–1	0	
b	9	2	0	1/2	1/2	1	
		Netevaluation	0	–9/2	–3/2	0	

Optimalsolutionis*b*=2andProfitis2×9=Rs.18/–

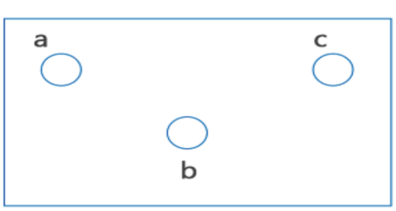
Q15. Write short notes on the following:

(i) Null graph

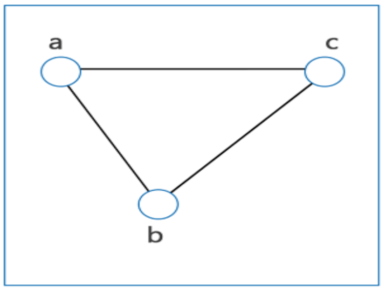
(ii) Simple graph

(iii) Multi graph

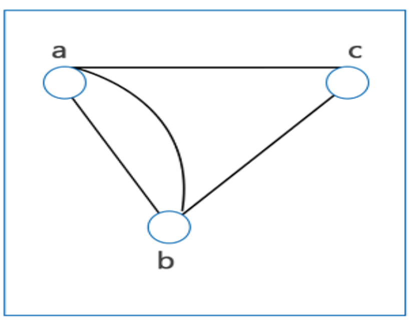
Ans.(i) Null graph- A graph will be known as the null graph if it contains no edges. With the help of symbol Nn, we can denote the null graph of n vertices. The diagram of a null graph is described as follows.



(ii) Simple graph- A graph will be known as a simple graph if it does not contain any types of loops and multiple edges. The simple graph must be an undirected graph. The diagram of a simple graph is described as follows:

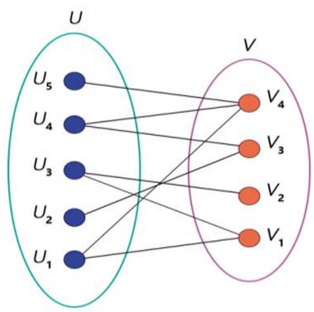


(iii) Muli graph- A graph will be known as a multi-graph if the same sets of vertices contain multiple edges. In this type of graph, we can form a minimum of one loop or more than one edge. The diagram of multi-graph is described as follows:



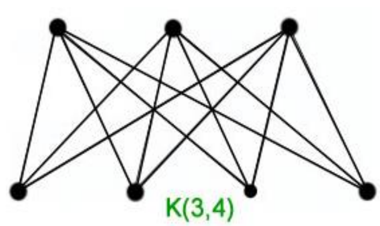
Q16. Define bipartite graph & complete bipartite graph.

Ans. Bipartite graph- A simple graph will be known as the bipartite graph if there are two independent sets which contain the set of vertices. The vertices of this graph will be connected in such a way that each edge in this graph can have a connection from the first set to the second set. That means the vertices of a first set can only connect with the vertices of a second set. Similarly, the vertices of a second set can only connect with the vertices of a first set. But this graph does not contain any edge which can connect the vertices of same set. The diagram of a bipartite graph is described as follows:



In the above graph, we have two sets of vertices. The Set U contains 5 vertices, i.e., U1, U2, U3, U4, U5, and the set V contains 4 vertices, i.e., V1, V2, V3, and V4. The vertices of set U only have a mapping with vertices of set V. Similarly, vertices of set V have a mapping with vertices of set U. Set U and set V does not have a connection to the same set of vertices. So this graph is a bipartite graph.

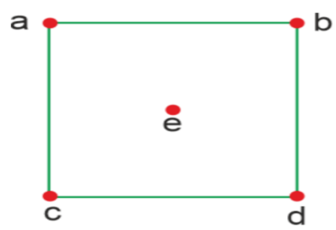
Complete bipartite graph - A graph will be known as the complete bipartite graph if it contains two sets in which each vertex of the first set has a connection with every single vertex of the second set. With the help of symbol K(X, Y), we can indicate the complete bipartite graph. That means the first set of the complete bipartite graph contains the x number of vertices and the second graph contains number of verticesthe y.



In the above graph, there are a total of two sets. The first set contains the 3 vertices, and the second set contains the 4 vertices. That means the value of x, y will be 3, 4. Every vertex of the first set has a connection with every vertex of a second set. So this graph is a complete bipartite graph.

Q17. What is degree of vertex?

Ans. In any graph, the degree can be calculated by the number of edges which are connected to a vertex. The symbol deg(v) is used to indicate the degree where v is used to show the vertex of a graph. So basically, the degree can be described as the measure of a vertex.



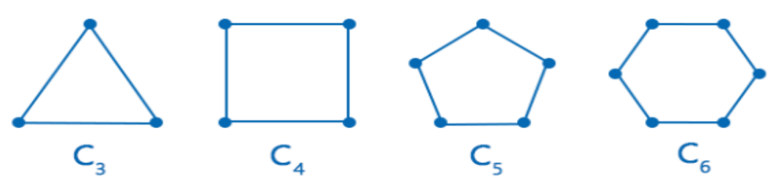
In the above graph, there are total of 5 vertices. The degree of vertex a is 2, the degree of vertex b is 2, the degree of vertex c is 2, the degree of vertex d is 2, and the degree of vertex e is zero.

Q18. Write a short note on cycle.

Ans. Cycle: In any graph, a cycle can be described as a closed path that forms a loop. A cycle will be formed in a graph if there is the same starting and end vertex of the graph, which contains a set of vertices. A cycle will be known as a simple cycle if it does not have any repetition of a vertex in a closed circuit. With the help of symbol Cn, we can indicate the cycle graph. The cycle graph can be of two types, i.e., Even cycle and Odd cycle.

- Even cycle: If a graph contains the even number of nodes and edges in a cycle, then that type of cycle will be known as an even cycle.
- Odd cycle: If a graph contains the odd number of nodes and edges in a cycle, then that type of cycle will be known as an odd cycle.

The diagram of a cycle is described as follow



In the above graph, all the graphs have formed a loop, and if we start from any vertex, then we will be able to end the loop of the same vertex. That means in all the above graphs, the starting and end vertex is the same. So this graph is a cycle.

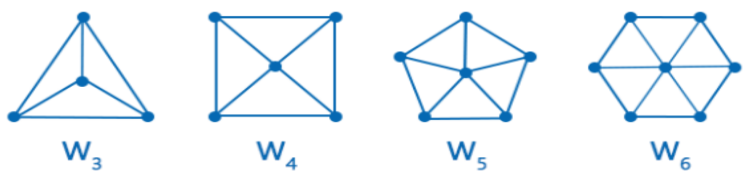
Graph C3 and C5 contain the odd number of vertices and edges, i.e., C3 contains 3 vertices and edges, and graph C5 contain 5 vertices and edges. So graphs C3 and C5 contain the odd cycle. Similarly, graph C4 and C6 contain the even number of vertices and edges, i.e., C4 contain the 4 vertices and edges, and graph C6 contains the 6 vertices and edges. So graphs C4 and C6 contain the even cycle

Q19. What is wheel?

Ans. Wheels: A wheel and a circle are both similar, but the wheel has one additional vertex, which is used to connect with every other vertex. With the help of symbol Wn, we can indicate the wheels of n vertices with 1 additional vertex. In a wheel graph, the total number of edges with n vertices is described as follows:

2*(n-1)

The diagram of wheels is described as follows:



In the above diagram, we have four graphs W3, W4, W5, and W6. All the graphs have an additional vertex which is used to connect to all the other vertices. So these graphs are the wheels.

Q20. Write difference between directed and undirected graphs.

Ans. Directed Graph- In graphtheory, a directed graph is a graph made up of a set of vertices connected by edges, in which the edges have a direction associated with them.

Undirected Graph- The undirected graph is defined as a graph where the set of nodes are connected together, in which all the edges are bidirectional. Sometimes, this type of graph is known as the undirected network.

Q21.What is a graph theory?

Ans. A graph theory is a study of graphs in discrete mathematics. The graphs here are represented by vertices (V) and edges (E). A graph here is symbolised as G(V, E).

Q22.What is a finite graph?

Ans. A graph that has finite number of vertices and edges is called finite graph.

Q23.How many edges does a null graph have?

Ans. A null graph has no edges.

Q24.If the degree of the vertex is 2, then what vertex it is?

Ans. If the degree of vertex is 2, then it is an even vertex.

Q25.A simple graph is directed or undirected?

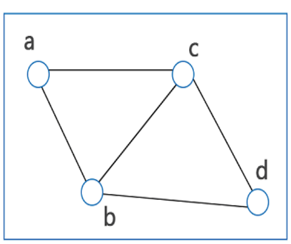
Ans. A simple graph is undirected and does not have multiple edges.

Q26.If two edges of a graph are connected by a single vertex, they are called adjacent edges. True or False?

Ans. True

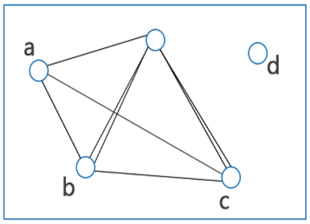
Q27. Write a short note on planer and non-planer graph.

Ans. **Planer Graph:** A graph will be known as the planer graph if it is drawn in a single plane and the two edges of this graph do not cross each other. In this graph, all the nodes and edges can be drawn in a plane. The diagram of a planer graph is described as follows:



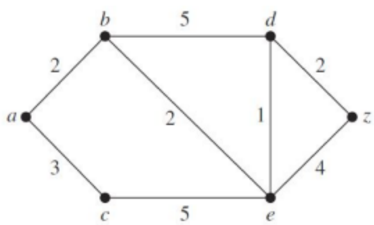
In the above graph, there is no edge which is crossed to each other, and this graph forms in a single plane. So this graph is a planer graph.

Non-planer graph: A given graph will be known as the non-planer graph if it is not drawn in a single plane, and two edges of this graph must be crossed each other. The diagram of a non-planer graph is described as follows:



In the above graph, there are many edges that cross each other, and this graph does not form in a single plane. So this graph is a non-planer graph.

Q28. Find a shortest path between a to z in the following graph.



Ans.

a	b	c	d	e	z
0 -	∞	∞	∞	∞	∞
0	2 -	3	∞	∞	∞
0	2	3 -	7	4	∞
0	2	3	7	4 -	∞
0	2	3	5 -	4	8
0	2	3	5	4	7-