Biyani's Think Tank

Concept based notes

Mechanics & Oscillations

(BSC Semester-I)

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Preface

I am glad to present this book, especially designed to serve the needs of the students. The book has been written keeping in mind the general weakness in understanding the fundamental concepts of the topics. The book is self-explanatory and adopts the "Teach Yourself" style. It is based on question-answer pattern. The language of book is quite easy and understandable based on scientific approach.

Any further improvement in the contents of the book by making corrections, omission and inclusion is keen to be achieved based on suggestions from the readers for which the author shall be obliged.

I acknowledge special thanks to Mr. Rajeev Biyani, Chairman & Dr. Sanjay Biyani, Director (Acad.) Biyani Group of Colleges, who are the backbones and main concept provider and also have been constant source of motivation throughout this endeavor. They played an active role in coordinating the various stages of this endeavor and spearheaded the publishing work.

I look forward to receiving valuable suggestions from professors of various educational institutions, other faculty members and students for improvement of the quality of the book. The reader may feel free to send in their comments and suggestions to the under mentioned address.

Author

Syllabus

S.No.	Units	Topics
1	**Unit–I** **Physical Law and frame of Reference**	**(a)Inertial and non- inertial frames,** Transformation of displacement, velocity. Acceleration between different frames of reference involving translation. Galilean transformation and invariance of Newton's laws. **(b) Coriolis Force:** Transformation of displacement, velocity and acceleration between rotating frame, Pseudo forces, Coriolis force, Motion relative to earth, Focult's pendulum. **(c) Conservative Forces:** Introduction about conservative and non-conservative forces, Rectilinear motion under conservative forces, Discussion of potential energy curve and motion of a particle.
2	**Unit– II** **Centre of Mass & Rigid body**	**Centre of Mass:** Introduction about Centre of Mass. Centre of Mass Frame: Collision of two particles in one and two dimensions (elastic and inelastic). Slowing down of neutrons in a moderator, Motion of a system with varying mass, Angular momentum concept, conservation and charge particle scattering by a nucleus. **Rigid body:** Equation of a motion of a rotating body. Inertial coefficient. Case of I not parallel to W. The kinetic energy of rotation and the idea of principal axes. The processional motion of the spinning Top
3	**Unit-Ill** **Motion under Central Forces & Damped Harmonic Oscillations**	**Motion under Central Forces:** Introduction about Centrul Forces, Motion under central forces, gravitational interaction. inertia und gravitational mass, General solution under gravitational interaction. Keplers laws, Discussion of trajectories, Cases of elliptical and circular orbits, Rutherford scattering. **Damped Harmonic Oscillations:** Introduction about oscillations in a potential well, Damped force and motion under damping. Damped Simple Harmonic Oscillator, Power dissipation, An harmonic oscillator and simple pendulum as an example.
4	**Unit-IV** **Driven Harmonic Oscillations & Coupled Oscillations**	**Driven harmonic oscillator: Driven** harmonic oscillator with damping. Frequency response. Phase factor, Resource,. Seriau and parallel of LCR circuit, Electromechanical Galvanometer. **Coupled Oscillations:** Equation of motion of two coupled Simple Harmonic Oscillators. Normal modest motion in mixed modes. Trust behavior, Dynamics of a number of oscillators with neighbor interactions.

Short Questions:

Qes.1. Define frame of reference?

Ans. A frame of reference is a well-defined coordinate system, and with respect to this the state of rest or the motion of a body is described. There are two types of frames of references, they are (i) inertial or non-accelerating frames and (ii) non-inertial or accelerating frames.

Qes.2. What is Galilean transformations?

Ans. Galilean transformations, also called Newtonian transformations, set of equations in classical physics that relate the space and time coordinates of two systems moving at a constant velocity relative to each other.

Qes.3. What is Foucault pendulum?

Ans. It is a device which demonstrate that earth rotates from west east about its own axis and take 24 hours.

Qes.4. Define conservative and non-conservative forces?

Ans. A conservative force is one for which the work done is independent of path. Equivalently, a force is conservative if the work done over any closed path is zero. A non-conservative force is one for which the work done depends on the path.

Qes.5.What is meant by centre of mass?

Ans. The center of mass is a position defined relative to an object or system of objects. It is the average position of all the parts of the system, weighted according to their masses.

Qes.6 Write the working principle of a Rocket?

Ans. Conservation of momentum.

Qes.7 Define the moment of inertia?

Ans. Moment of inertia is defined as the quantity expressed by the body resisting angular acceleration which is the sum of the product of the mass of every particle with its square of a distance from the axis of rotation

Qes.8 Define the inertial mass?

Ans.The ratio of force over acceleration is called inertial mass. Inertial mass is a measure of how difficult it is to change the velocity of an object.

Qes.9 Write the Kepler'slaw of planetarymotion ?

Ans.According to Kepler's law of periods," The square of the time period of revolution of a planet around the sun in an elliptical orbit is directly proportional to the cube of its semi-major axis"

Qes. 10 Define Simple harmonic motion?

Ans. S.H.M:- If the acceleration of the vibrating body directly varies with the displacement of the body from the mean position and always directed to the mean position, the motion of that body is called simple harmonic motion. Ex:- (i) The motion of a pendulum is an S.H.M.

Qes.11Write the general equation of damped harmonic motion?

Ans. $md^2x/dt^2 + bdx/dt + kx = 0$.

Qes.12.What is meant by coupled Oscillators?

Ans. Coupled Oscillations occur when two or more oscillating systems are connected in such a manner as to allow motion energy to be exchanged between them.

Qes.13What do you meant by quality factor?

Ans. Quality factor or Q factor is a dimensionless parameter that describes how under damped an oscillator or resonator is. It is defined as the ratio of the initial energy stored in the resonator to the energy lost in one radian of the cycle of oscillation.

Qes.14 What are driven harmonic Oscillations?

Ans. A harmonic oscillator is a system in which an object vibrates with a certain amplitude and frequency. In a simple harmonic oscillator the are no external forces, such as friction or driving forces working on the object.

Qes.15 In forced Oscillator, the energy transfer to oscillator by driven force will maximum at__?

Ans. Resonance

ANSWER BRIEFLY:

Qes.1 Prove that a reference frame moving with constant velocity with respect to an inertial frame is also inertial frame?

Ans.

Solution : Let S is an inertial frame of reference and another frame S′ moving with constant velocity $\vec{v}$ with resepct to S. Let at t = 0 the origin (O and O′) of both frames coincide. So at any time t, the position vector

[redacted]

Qes.2 Prove that the acceleration of particle remains invariant under Galilean transformation?

Ans.

If S′ is moving relative to S with constant velocity $\vec{V}$ in any direction then

$$\vec{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k} \quad .....(1.34)$$

and displacement of O′ relative to O at any point of time t will be OO′ = $\vec{V}t$ if the position vector of particle P in S and S′ are $\vec{r}$ and $\vec{r}'$ respectively.

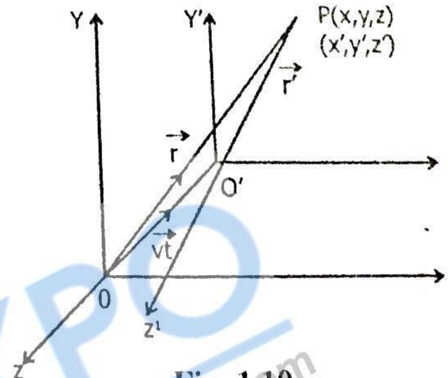


Fig. 1.10

then $$\vec{r} = \vec{r}' + OO'$$

$$= \vec{r}' + \vec{V}t \quad .....(1.35)$$

or $$\vec{r}' = \vec{r} - \vec{V}t \quad .....(1.36)$$

In term of component

$$\left.\begin{aligned} x' &= x - V_x t \\ y' &= y - V_y t \\ z' &= z - V_z t \end{aligned}\right\} \quad .....(1.37)$$

above equation is Galilean transformation equation of co-ordinates.

$$\frac{d\vec{r}'}{dt} = \frac{d\vec{r}}{dt} - \vec{V}$$

or $$\vec{v}' = \vec{v} - \vec{V} \quad .....(1.38)$$

Where $\vec{v}$ and $\vec{v}'$ are velocity of particle in frame S and S′ respectively. The above equation is transformation equation of velocity which proves that velocity is not invariant in this case also.

Transformation of acceleration : differentiating equation (1.38) w.r.t. to time

$$\frac{d\vec{v}'}{dt} = \frac{d\vec{v}}{dt} - \frac{d\vec{V}}{dt} \qquad V \text{ is constant} \quad \therefore \frac{d\vec{V}}{dt} = 0$$

$$\therefore \quad \vec{a}' = \vec{a} \quad .....(1.39)$$

Here a and a’ are acceleration of particle P in frame S and S’ respectively.

Qes.3 Find the effect of coriolis force on a freely falling body?

Ans. A body ofmass m is falling freely in gravitational force a height λ from a heigh N latitude .then the velocity of body in z axis or in direction of –k' unit vector

$$\vec{v}' = -v'\hat{k}'$$

The component of angular velocity $\vec{\omega}$ of earth

$$\vec{\omega} = \omega\cos\hat{j}' + \omega\sin\lambda\,\hat{k}'$$

so the coriolis force on body

$$\vec{F}_C = -2m(\vec{\omega}\times\vec{v}')$$

$$= -2m[(\omega\cos\lambda\,\hat{j}' + \omega\sin\lambda\,\hat{k}')\times(-v')\hat{k}']$$

$$= -2m\,\omega(-v'\cos\lambda\,\hat{i}')$$

$$= 2m\,\omega\,v'\cos\lambda\,\hat{i}'$$

Since the direction of force is x' in east, so the body falling parpendicular in North zone is shifted dowards East.

From Newton's second law the equation of motion of body in x'-axis is

$$m\frac{d^2x'}{dt^2} = 2m\,\omega v'\cos\lambda$$

Since the particle is falling parpendicular downwards. So at time t the vertical velocity of particle is $v' = gt$

$$\frac{d^2x'}{dt^2} = 2g\,\omega t\cos\lambda$$

Take integration of equation

$$\frac{dx'}{dt} = 2g\,\omega\frac{t^2}{2}\cos\lambda + C_1$$

Here C_1 is integral constant.

Initialy at $t = 0$, $\frac{dx'}{dt'} = 0$ so $C_1 = 0$

$$\therefore \quad \frac{dx'}{dt} = g\,\omega t^2\cos\lambda$$

$$x' = \frac{1}{3} g \omega t^3 \cos\lambda + C_2$$

Here C_2 is integral constant.

When particle starts falling there is no displacement of particle in East.

$\therefore$ at $t = 0$

$x' = 0$

so $C_2 = 0$

$$\therefore \quad x' = \frac{1}{3} g \omega t^3 \cos\lambda$$

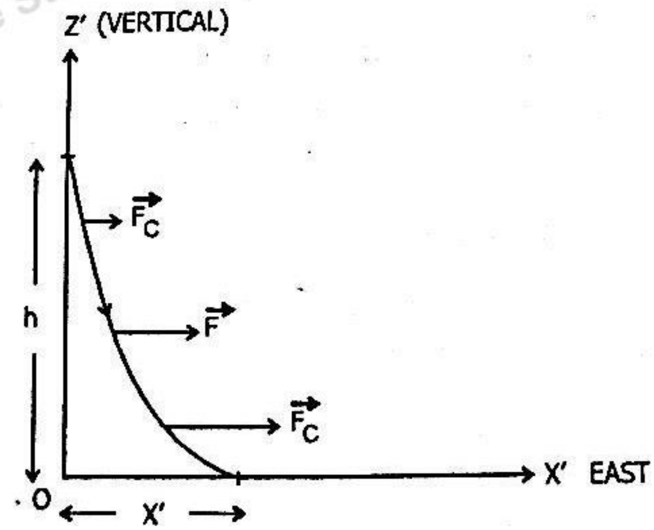
equation gives displacement (in x'-axis) of body falling verticaly. In north zone the deviation is in East direction and in sourth zone t he deviation is in west direction. If the body is falling from a height h then

$$h = \frac{1}{2} g t^2$$

$$\therefore \quad t = \sqrt{\left(\frac{2h}{g}\right)}$$

$$x' = \frac{1}{3} g \omega \cos\lambda \left(\frac{2h}{g}\right)^{3/2}$$

$$= \frac{2}{3} h \omega \cos\lambda \sqrt{\left(\frac{2h}{g}\right)}$$



Qes.4 Describe rectilinear motion of a particle under conservative force field.

Consider a particle of mass m is moving along x-direction in rectilinear motion in conservative force field. As the force is conservative so its value depends only on position of particle and potential energy of this parlicle also depends on position-x of the particle so total energy of particle

$$E = \frac{1}{2}mv^2 + U(x) \quad .....(4.56)$$

Where v is velocity or particle and U(x) is potential energy of particle. According to conservation of energy E is constant. Because motion is rectilinear, So velocity

$$v = \frac{dx}{dt} \quad .....(4.57)$$

or from equation (4.56)

$$v = \frac{dx}{dt} = \sqrt{\frac{2[E - U(x)]}{m}}, \quad .....(4.58)$$

or

$$\frac{dx}{\left[\frac{2}{m}[E - U(x)]\right]^{1/2}} = dt, \quad .....(4.59)$$

Integration from initial time of equation (4.56) $t_0 = 0$ (where position of particle is x_0) to current time t (where position of particle is x)

$$= \int_{x_0}^{x} \frac{dx}{\left[\frac{2}{m}\{E - U(x)\}\right]^{1/2}} = \int_0^t dt$$

or

$$t = \int_{x_0}^{x} \frac{dx}{\left[\frac{2}{m}\{E - U(x)\}\right]^{1/2}} \quad .....(4.60)$$

Solution of above equation by putting the value of U(x), we can find the relation between position of particle x and time t.

If given that conservative force is constant, F = constant, So potential energy

$$U = -\int F dx = -Fx + C$$

if potential energy is zero at x = 0 then C = 0

So $\quad U = -Fx \quad .....(4.61)$

Using this in equation (4.60)

$$t = \frac{1}{\sqrt{(2/m)}} \int_0^x \frac{dx}{\sqrt{(E + Fx)}},$$

Where E and F are constants. So by integration

$$E = \frac{1}{2} m v_0^2,$$

or $$\sqrt{\left(\frac{2E}{m}\right)} = v_0,$$

So in a constant conservative force field, relation between position x of moving particle in rectilinear motion and time t is

$$x = \frac{1}{2} a t^2 + v_0 t \qquad .....(4.63)$$

This equation explain the rectilinear motion of particle for any conservative force.

Qes.5 If Q is the quality factor then prove that frequency of oscillator reduces 12.5/Q^2 % due to damping ?

Ans

$$\omega = \sqrt{\omega_o^2 - \gamma^2}$$

or $$\omega^2 = \omega_o^2 - \gamma^2$$

or $$\frac{\omega^2}{\omega_o^2} = 1 - \frac{\gamma^2}{\omega_o^2}$$

$\because$ $$Q \approx \frac{\omega_o}{2\gamma}$$

$\Rightarrow$ $$\gamma = \frac{\omega_o}{2Q}$$

Putting the value of γ in equation (i)

$$\frac{\omega^2}{\omega_o^2} = 1 - \frac{1}{4Q^2}$$

or $$\frac{\omega}{\omega_o} = \left(1 - \frac{1}{4Q^2}\right)^{\frac{1}{2}}$$

$$\frac{\omega}{\omega_o} \cong 1 - \frac{1}{8Q^2} \qquad \because Q >> 1$$

or $$1 - \frac{\omega}{\omega_o} = \frac{1}{8Q^2}$$

or $$\frac{\omega_o - \omega}{\omega_o} \times 100 = \frac{100}{8Q^2}$$

Qes.6 Explain relation between angular moment and torque?

Ans. Rate of change of angular momentum is equal to torque.

Following is the derivation for this relation.

$$\mathbf{L} = \sum_{i=1}^{n} (\mathbf{r}_i \times \mathbf{p}_i)$$

Differentiating above equation with respect to time t we get

$$\frac{d\mathbf{L}}{dt} = \frac{d}{dt}\left[\sum_{i=1}^{n} (\mathbf{r}_i \times \mathbf{p}_i)\right] = \sum_{i=1}^{n}\left[\frac{d\mathbf{r}_i}{dt} X \mathbf{p}_i + \mathbf{r}_i X \frac{d\mathbf{p}_i}{dt}\right]$$

But $\frac{d\mathbf{r}_i}{dt} = \mathbf{v}_i$ and $\mathbf{p}_i = m\mathbf{v}_i$

Hence $\frac{d\mathbf{r}_i}{dt} \times \mathbf{p}_i = \mathbf{v}_i \times m\mathbf{v}_i = m(\mathbf{v}_i \times \mathbf{v}_i) = 0$

And $\frac{d\mathbf{p}_i}{dt} = \mathbf{F}_i$

$$\therefore \frac{d\mathbf{L}}{dt} = \sum_{i=1}^{n} (\mathbf{r}_i \times \mathbf{F}_i) \qquad (2)$$

If the torque acting on the system of particles is τ then

$$\tau = \frac{d\mathbf{L}}{dt} = \sum_{i=1}^{n} (\mathbf{r}_i \times \mathbf{F}_i) \qquad (3)$$

Qes.7 Derive expression for the moment of inertia of solid sphere about its diameter?

Ans According to the figure a sphere of mass M and radius R is shown, whose density is p. We have to calculate the moment of inertia of the sphere about the diameter XX'. We can assume the sphere to be made up of many discs whose surfaces are parallel to YY' and the center is on XX' axis. One of these discs has a center at O' and radius y; and the distance of the O' circle from center O is x; the width of this disc is dx.

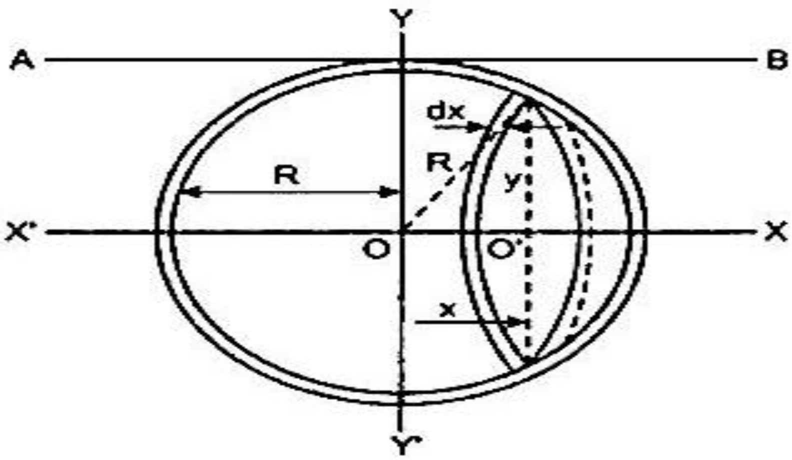


Fig: Moment of Inertia of a solid sphere about its diameter

Density of the sphere (ρ) = $\frac{M}{\frac{4}{3}\pi R^3}$ .... (1)

Volume of the disc = $\pi y^2 dx$

and the mass of the disc = $\pi y^2 dx\, \rho$ .... (2)

Therefore, the moment of inertia of the sphere about the axis XX' perpendicular to the surface (plane) and passing through the center is;

$$dI = \frac{1}{2} dm y^2$$

$$= \frac{1}{2}(\pi y^2\, dx.\rho)\, y^2$$

(putting the value of *dm* from equation 2)

$$= \frac{1}{2}\pi y^4 \rho\, dx$$

$$= \frac{1}{2}\pi (R^2 - x^2)\rho\, dx \quad (\because y^2 = R^2 - x^2)$$

The moment of inertia of the total sphere about the XX' axis will be equal to the sum of the moment of inertia of all the discs between x = -R and x = +R.

$$\textbf{Therefore; } I = \int dI = \int_{-R}^{R} \frac{1}{2}\pi (R^2 - x^2)^2 \rho\, dx$$

$$= 2 \times \frac{1}{2}\pi\rho \int_0^R (R^2 - x^2)^2\, dx$$

$$= \pi\rho \int_0^R (R^4 + x^4 - 2R^2x^2)\, dx$$

$$= \pi\rho\int_0^R (R^4 + x^4 - 2R^2x^2)\,dx$$

$$= \pi\rho\left[R^4x + \frac{x^5}{5} - \frac{2R^2x^3}{3}\right]_0^R$$

$$= \pi\rho\left[R^5 + \frac{R^5}{5} - \frac{2R^5}{3}\right]$$

$$= \pi\rho\left[\frac{8}{15}R^5\right]$$

$$= \frac{\pi 3M}{4\pi R^3} \times \frac{8}{15}R^5 \qquad \text{(From equation 1)}$$

$$\boxed{I = \frac{2}{5}MR^2} \qquad \ldots(3)$$

Qes.8 If the average distance of planet mars from sun is 1.5234 times the distance of earth from sun, then calculate the time period of marse around sun?

Ans.It is given

$$\frac{a_m}{a_E} = \frac{\text{average distance of planet mars from sun}}{\text{average distance of earth from sun}}$$

$$\frac{a_m}{a_E} = 1.524$$

From Kepler's third law

$$T_M^2 \propto a_m^3 \quad \text{and} \quad T_E^2 \propto a_E^3$$

Hence $\quad \dfrac{T_m^2}{T_E^2} = \dfrac{a_m^3}{a_E^3} \Rightarrow (1.524)^{3/2} = \dfrac{T_m}{T_E}$

or Time period of mars revolution

$$T_M = (1.524)^{3/2} \times T_E$$

$$= (1.524)^{3/2} \times 365$$

$$= 686.7 \text{ Days}$$

Qes.9 Drive the equation of motion of a particle under the influence of central force?

According the property of the field of central force the motion of the particle is in some plane. Let this motion is in XY plane. Assume a particle of mass "m" moving under this force which have a vector position $\vec{r}$ and Polar coordinates (r, θ).

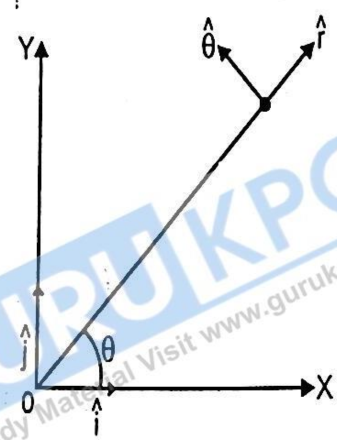


Fig. 7.1

So, $$\vec{r} = r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j} \qquad .....(7.7)$$

In polar coordinates, $\hat{r}$ and $\hat{\theta}$ are unit vectors, which are perpendicular to $\vec{r}$ and $\vec{r}$ respectively (In a direction for increasing θ).

They are respectively

$$\hat{r} = \frac{\partial\vec{r}/\partial r}{|\partial\vec{r}/\partial r|} \text{ and } \hat{\theta} = \frac{\partial\vec{r}/\partial\theta}{|\partial\vec{r}/\partial\theta|} \qquad .....(7.8)$$

defined as written here.

This can be prove by using equations (7.7) and (7.8), that

$$\hat{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j} \quad .....(7.9a)$$

or

$$\hat{\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j} \quad .....(7.9b)$$

So, in polar coordinates the rate of change of these unit vectors with respect to time are respectively

$$\frac{d\hat{r}}{dt} = \frac{\partial\hat{r}}{\partial r}\frac{\partial r}{\partial t} + \frac{\partial\hat{r}}{\partial\theta}\frac{\partial\theta}{\partial t}$$

$$= 0\cdot\frac{\partial r}{\partial t} + (-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})\frac{\partial\theta}{\partial t}$$

$$\boxed{\frac{d\hat{r}}{dt} = \hat{\theta}\frac{\partial\theta}{\partial t}} \quad .....(7.10)$$

or

$$\frac{\partial\hat{\theta}}{\partial t} = \frac{\partial\hat{\theta}}{\partial r}\frac{\partial r}{\partial t} + \frac{\partial\hat{\theta}}{\partial\theta}\frac{\partial\theta}{\partial t}$$

$$= 0\cdot\frac{\partial r}{\partial t} + (-\cos\theta\,\hat{i} - \sin\theta\,\hat{j})\frac{\partial\theta}{\partial t}$$

$$\boxed{\frac{\partial\hat{\theta}}{\partial t} = -\hat{r}\frac{\partial\theta}{\partial t}} \quad .....(7.11)$$

Using equations (7.10) and (7.11), the velocity and acceleration of the particle can be find as

Velocity $$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d(r\,\hat{r})}{dt} = \frac{dr}{dt}\hat{r} + r\frac{d\hat{r}}{dt}$$

$$\vec{v} = \frac{dr}{dt}\hat{r} + r\frac{d\theta}{dt}\hat{\theta} \quad .....(7.12)$$

Thus, $$\vec{v} = \vec{v}_r + \vec{v}_\theta \quad .....(7.13)$$

Can be written, where the radial and angular component of velocity are expressed as,

$$\vec{v}_r = \frac{dr}{dt}\hat{r} \quad .....(7.14)$$

or

$$\vec{v}_\theta = r\frac{d\theta}{dt}\hat{\theta} \quad .....(7.15)$$

Differentiating equation (7.12) with respect to time gives the equation of acceleration of the particle, which is

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[\frac{dr}{dt}\hat{r} + r\frac{d\theta}{dt}\hat{\theta}\right]$$

$$= \frac{d^2r}{dt^2}\hat{r} + \frac{dr}{dt}\frac{d\hat{r}}{dt} + \frac{dr}{dt}\frac{d\theta}{dt}\hat{\theta} + r\frac{d^2\theta}{dt^2}\hat{\theta} + r\frac{d\theta}{dt}\frac{d\hat{\theta}}{dt}$$

After putting the value of $\frac{d\hat{r}}{dt}$ and $\frac{d\hat{\theta}}{dt}$ from equations (7.10) and (7.11), we get

$$\vec{a} = \frac{d^2r}{dt^2}\hat{r} + \frac{dr}{dr}\hat{\theta}\frac{d\theta}{dt} + \frac{dr}{dt}\frac{d\theta}{dt}\hat{\theta} + r\frac{d^2\theta}{dt^2}\hat{\theta} + r\frac{d\theta}{dt}\left(-\hat{r}\frac{d\theta}{dt}\right)$$

$$\vec{a} = \left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]\hat{r} + \left[r\frac{d^2\theta}{dt^2} + \frac{2dr}{dt}\frac{d\theta}{dt}\right]\hat{\theta} \qquad .....(7.16)$$

This $\vec{a}$ can also be expressed in the form of radial component $\vec{a}_r$ and angular component $\vec{a}_\theta$

So, $$\vec{a} = \vec{a}_r + \vec{a}_\theta \qquad .....(7.17)$$

where $$\vec{a}_r = \left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right] \qquad .....(7.18)$$

or $$\vec{a}_\theta = \left[r\frac{d^2\theta}{dt^2} + \frac{2dr}{dt}\frac{d\theta}{dt}\right] \qquad .....(7.19)$$

The force acting on the particle according to the Newton's second law of motion

$$\vec{F} = m\vec{a}$$

Thus, $$\vec{F} = m(\vec{a}_r + \vec{a}_\theta) = m\vec{a}_r + m\vec{a}_\theta \qquad .....(7.20)$$

Since central force is a function of $\vec{r}$ only

$\therefore$ $$F(r)\hat{r} = m\vec{a}_r + m\vec{a}_\theta$$

$$F(r)\hat{r} = ma_r\hat{r} + ma_\theta\hat{\theta}$$

On compairing the coefficients of $\vec{r}$ and $\hat{\theta}$ in above equation and using equation (7.18) and equation (7.19). The following equation of motion is obtained

$$m\left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right] = F(r) \qquad .....(7.21)$$

or $$m\left[r\frac{d^2\theta}{dt^2} + 2\frac{d\theta}{dt}\cdot\frac{d\theta}{dt}\right] = 0 \qquad .....(7.22)$$

Which are known as the equations of motion of the particle under central force in polar co-ordinates. These equations help out to explain the law of conservation of angular momentum and constancy of Areal velocity.

Qes.10 Describe the differential equation for damped harmonic motion ?

Ans.

In simple harmonic motion, the restoring force is directly proportional to the displacement and acts in the direction opposite to the displacement.

$\Rightarrow \quad F = ma = m\frac{d^2x}{dt^2} = -kx,$ where k is the constant of proportionality.

In real life, there is always some frictional force (or deliberate damping) which also opposes the motion. Quite often, the damping force is proportional to the velocity.

$\Rightarrow \quad m\frac{d^2x}{dt^2} = -kx - c\frac{dx}{dt},$ where c is the constant of proportionality.

$\Rightarrow \quad \frac{d^2x}{dt^2} + \frac{c}{m}\frac{dx}{dt} + \frac{k}{m}x = 0.$

This can be written as, $\frac{d^2x}{dt^2} + 2\zeta\omega\frac{dx}{dt} + \omega^2 x = 0,$ where,

$\omega = \sqrt{\frac{k}{m}}$ and $\zeta = \frac{c}{2\sqrt{mk}}.$

This is the differential equation of a damped harmonic motion.

Qes.11 Write differential equation of motion of a driven oscillator?

Ans. To find the one dimensional equation of motion of driven harmonic oscillator.We assume an oscillator of mass 'm'. On which the resultant force $F_a$is acting due to viscous force in the air medium and frictional force. This resultant force is proportional to the velocity of the oscillator and is in the direction opposite to the direction of motion of the oscillator due to which the amplitude of an oscillator does not remain constant with time and due to which the motion of an oscillator is not simple harmonic.

In this condition due to external driven force, the decay of energy is compensated and the amplitude of an oscillator is kept constant. In fact the only way of maintaining the amplitude of a damped oscillator is to continuously feed energy into the system in such a manner as to offset the frictional loses. A steady (i.e. constant amplitude) oscillation of this type is called driven harmonic oscillation.

The three forces acting on the forced oscillator are

(i) Restoring force $F_R = -kx$

(ii) Damping force $F_d = -\lambda \frac{dx}{dt}$

(iii) Driven force $F_f = F_o \sin\omega t$

Thus, at some instant the displacement of an oscillator is "x" then the resultant force acting on the oscillator is

$$\vec{F} = \vec{F_R} + \vec{F_d} + \vec{F_f}$$

or $$\vec{F} = -kx - \lambda \frac{dx}{dt} + F_o \sin\omega t \quad .....(9.1)$$

From the Newton's law of motion, if the acceleration of oscillator of mass "m" is $\frac{d^2x}{dt^2}$, then, the resultant force on oscillator will be

$$\vec{F} = m\frac{d^2x}{dt^2} \quad .....(9.2)$$

Compare equation (9.1) and equation (9.2), we get

$$m\frac{d^2x}{dt^2} = -kx - \lambda \frac{dx}{dt} + F_o \sin\omega t$$

or $$\frac{d^2x}{dt^2} + \frac{\lambda}{m}\frac{k}{m} + \frac{dx}{dt}x = \frac{F_o}{m}\sin\omega t$$

or $$\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega_0^2 x = f_o \sin\omega t \quad .....(9.3)$$

when $\frac{\lambda}{m} = 2\gamma = \frac{1}{\tau}$ (damping constant)

$\sqrt{\frac{k}{m}} = \omega_0$ (Natural frequency of oscillator)

ω = frequency of driven harmonic force

$f_0 = \frac{F_0}{m}$, is a constant.

Equation 9.3 represents the generalized equation of motion of driven harmonic oscillator.

Qes.12 Comparison between mechanical and electrical forced oscillators?

Ans.

Electrical system	Mechanical system
Charge q	Displacement x
Current $i = \frac{dq}{dt}$	Velocity $v = \frac{dx}{dt}$
Inductance L	Mass m
Reciprocal of capacitance $\frac{1}{C}$	Force constant k
Electrical energy $= \frac{1}{2}\left(\frac{1}{C}\right)q^2$	Potential energy $= \frac{1}{2}k\,x^2$
Magnetic energy $= \frac{1}{2}Li^2$	Kinetic energy $= \frac{1}{2}mv^2$
Electromagnetic energy $U = \frac{1}{2}\left(\frac{1}{C}\right)q^2 + \frac{1}{2}Li^2$	Mechanical energy $E = \frac{1}{2}k\,x^2 + \frac{1}{2}mv^2$

158-I

B.Sc./B.Ed. (Four Year) (Part I) EXAMINATION, 2019

PHYSICS-I

(Mechanics)

Time Allowed : Three Hours | Maximum Marks: 3

समय 3 घंटे | अधिकतम अंक 3

No supplementary answer-book will be given to any candidate. Hence the candidate should write the answ precisely in the main answer-book only.

किसी भी परीक्षार्थी को पूरक उत्तर-पुस्तिका नहीं दी जायेगी। अतः परीक्षार्थियों को चाहिए कि वे मुख्य उत्तर-पुस्तिका ही समस्त प्रश्नों के उत्तर लिखें।

All the Parts of one question should be answered at one place in the answer-book. One complete question sh not be answered at different places in the answer-book.

किसी भी एक प्रश्न के अन्तर्गत पूछे गये विभिन्न प्रश्नों के उत्तर, उत्तर-पुस्तिका में अलग-अलग स्थानों पर हल के बजाय एक ही स्थान पर हल करें।

First question carries 9 marks and is compulsory. First question has six parts of short answer type. Othe

प्रश्नों के उत्तर लिखने से पूर्व प्रश्न-पत्र पर [illegible] अवश्य लिखें।

1

(a) Define inertial and non-inertial frames
जड़त्वीय एवं अजड़त्वीय निर्देश तंत्र की परिभाषा दीजिए।

(b) In how much time will the oscillation of a foucalt pendulum complete one turn when it

(i) at equator

(ii) at north pole

(iii) at 45° north latitude

कितने समय में एक फोको लोलक का दोलन तल एक चक्कर पूरा करेगा जबकि वह स्थित है

(i) भूमध्य रेखा पर

(ii) उत्तरी ध्रुव पर

(iii) 45° उत्तरी अक्षांश पर

(c) Define precessional motion with suitable example
पुरस्सरण गति को उदाहरण सहित परिभाषित कीजिए।

(d) Find the reduced mass of deutron. 1½

ड्यूट्रॉन का समानीत द्रव्यमान ज्ञात कीजिए।

(e) Write three Kepler's law of planetary motion 1½

कैप्लर के ग्रहीय गति के तीन नियम लिखिए।

(f) In an LCR circuit if L = 2MH, C = 2 F and R = 0.2 Ω. Calculate the resonance frequency and quality factor of the circuit. 1½

LCR परिपथ में L = 2MH, C = 2 F एवं R = 0.2 Ω हो, तो परिपथ की अनुनादी आवृत्ति एवं विशेषता गुणांक ज्ञात कीजिए।

UNIT-I/इकाई–I

(a) Show that the displacement of bodies falling vertically downward on earth is given by 1

$$x' = \frac{2}{3} h w \cos\lambda \sqrt{\frac{2h}{g}}$$

दिखाइए कि पृथ्वी पर ऊर्ध्वाधर नीचे की ओर गिरते हुए पिण्ड का विस्थापन निम्न से दिया जाता है।

$$x' = \frac{2}{3} h w \cos\lambda \sqrt{\frac{2h}{g}}$$

(b) Find the x, y, z components of the force when the body is at a position (–2, 0, 5). The potential energy is given by $u = 40 + 6x^2 - 7xy + 8y^2 + 32z$, where u is in Joule and x, y, z are in meters 3

किसी पिण्ड पर लगने वाले बल के x, y, z घटकों को ज्ञात कीजिए जबकि पिण्ड (–2, 0, 5) स्थिति पर है। स्थितिज ऊर्जा है – $u = 40 + 6x^2 - 7xy + 8y^2 + 32z$, जहाँ u जूल में है एवं x, y, z मीटर में है।

OR/अथवा

Prove that the force $\vec{F} = [(2xy + z^2)\hat{i} + x^2\hat{j} + 2xz\hat{k}]$ is conservative. Find the potential energy function of above force. 3

3\.

(a) Define Angular momentum and torque of a particle system. Derive expression for the relation between angular momentum and torque. 3

किसी कण तंत्र के कोणीय संवेग तथा बल–आघूर्ण को परिभाषित कीजिए। कोणीय संवेग एवं बलाघूर्ण के सम्बन्ध के लिए सूत्र व्युत्पन्न कीजिए।

(b) If the earth is suddenly contracted so that changed radius becomes half the present radius of the earth, then what will be the duration of a day? 3

यदि पृथ्वी सहसा इतनी संकुचित की जाये कि उसकी परिवर्तित त्रिज्या वर्तमान त्रिज्या की आधी रह जाये तो दिन की अवधि कितनी होगी?

OR/अथवा

A Circular disc of radius 0.5 meter and mass 25 kg is rotating about its own axis with a speed of 120 rev/min. Calculate the rotational kinetic energy of the disc.

एक वृत्ताकार चकती जिसकी त्रिज्या 0.5 मीटर एवं द्रव्यमान 25 किग्रा. है, अपनी धुरी पर 120 चक्कर / मिनट की रफ्तार से घूर्णन रफ्तार से घूर्णन करती है। चकती के घूर्णन की गतिज ऊर्जा परिकलन कीजिए।

UNIT–III /इकाई–III

4\.

(a) When a particle moves under the influence of a central force, prove that 3

(i) Angular momentum of a particle remains conserved.

(ii) Total energy of the particle remains conserved.

सिद्ध कीजिए कि जब कोई कण केन्द्रीय बल के प्रभाव में गति करता है तो

(i) कण का कोणीय संवेग संरक्षित रहता है।

(ii) कण की कुल ऊर्जा सदैव नियत रहती है।

(b) Prove that the displacement average kinetic and potential energy of simple harmonic oscillator are $1/3\ MW_0^2a^2$ and $1/6\ MW_0^2a^2$ 3

सिद्ध कीजिये कि स्थिति के सापेक्ष आवर्ती दोलक की माध्य गतिज ऊर्जा व स्थितिज ऊर्जा क्रमशः $1/3\ MW_0^2a^2$ व $1/6\ MW_0^2a^2$ होती है।

OR/अथवा

If Q is the quality factor then prove that frequency of oscillator reduces to $12.5/Q^2$ % due to damping.

यदि Q विशेषता गुणक हो, तो सिद्ध करो कि अवमन्दन के कारण किसी दोलक की आवृत्ति $12.5/Q^2$ % कम हो जाती है।

UNIT–IV/इकाई–IV

5.

(a) Obtain an expression for power absorption by driven or forced oscillator. Also find the expression for quality factor of forced harmonic oscillator. 3

चालित या प्रणोदित दोलक के शक्ति अवशोषण तथा विशेषता गुणांक का व्यंजक प्राप्त कीजिए।

(b) Amplitude of oscillation of driven oscillator of 10 gm manss at low frequency is 0.01cm. At frequencies 512 Hz, it is increased to 1cm. Find the quality factor Q and damping coefficient λ. https://www.uoronline.com 3

10 gm के प्रणोदित दोलक का अत्यल्प आवृत्तियों का आयाम 0.01cm है। तथा यह बढ़कर 512 Hz अपमन्दन पर 1cm हो जाता है। दोलक का विशेषता गुणांक Q एवं अवमन्दन गुणांक λ ज्ञात कीजिए।

OR/अथवा

In an parallel LCR circuit if L = 1MH, C = 10 F, and R = 0.4Ω. Calculate quality factor Q.

एक समान्तर LCR परिपथ में यदि L = 1MH, C = 10 F, तथा R = 0.4Ω हो, तो विशेषता गुणांक Q का मान ज्ञात कीजिए।