Biyani's Think Tank

Concept based notes

Mathematical Methods for Numerical Analysis and Optimization

(BCA Part-II)

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Published by :
Hastlipi
"Geetanjali Apartment"
1926, Nataniyon ka Rasta
Jaipur-302 003 | Ph. : 0141-2317659

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Sole Distributor :
Shyam Prakashan
"Geetanjali Apartment"
1926, Nataniyon ka Rasta
Jaipur-302 003
Ph. : 0141-2317659
website : www.shyamprakashan.com
e-mail : ankit_146@sify.com

First Edition : 2009

Price : 90/-

Type Setted by :
Shri Padam Computer Centre, Jaipur

Printed :
Shital Printers, Jaipur

Preface

I am glad to present this book, especially designed to serve the needs of the students. The book has been written keeping in mind the general weakness in understanding the fundamental concept of the topic. The book is self-explanatory and adopts the "Teach Yourself" style. It is based on question-answer pattern. The language of book is quite easy and understandable based on scientific approach.

I have made a meaningful effort to summarize the complete syllabus. This includes various methods which are explained in simplest way.

Any further improvement in the contents of the book by making corrections, omission and inclusion is keen to be achieved based on suggestions from the reader for which the author shall be obliged.

I acknowledge special thanks to Mr. Rajeev Biyani, *Chiarman* & Dr. Sanjay Biyani, *Director (Acad.)* Biyani Group of Colleges, who is the backbone and main concept provider and also have been constant source of motivation throughout this endeavour. We also extend our thanks to M/s. Hastlipi, Omprakash Agarwal/Sunil Kumar Jain, Jaipur, who played an active role in co-ordinating the various stages of this endeavour and spearheaded the publishing work.

I look forward to receiving valuable suggestions from professors of various educational institutions, other faculty members and the students for improvement of the quality of the book. The reader may feel free to send in their comments and suggestions to the under mentioned address.

Author

Syllabus

B.C.A. Part-II

Mathematical Methods for Numerical Analysis and Optimization

Computer arithmetics and errors. Algorithms and programming for numerical solutions. The impact of parallel computer : introduction to parallel architectures. Basic algorithms Iterative solutions of nonlinear equations : bisection method, Newton-Raphson method, the Secant method, the method of successive approximation. Solutions of simultaneous algebraic equations, the Gauss elimination method. Gauss-Seidel Method, Polynomial interpolation and other interpolation functions, spline interpolation system of linear equations, partial pivoting, matrix factorization methods. Numerical calculus : numerical differentiating, interpolatory quadrature. Gaussian integration. Numerical solutions of differential equations. Euler's method. Runge-Kutta method. Multistep method. Boundary value problems : shooting method.

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Content

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Chapter-1

Computer Arithmetic and Errors

Q.1. An approximate value of π is given by x_1 = 22/7 = 3.1428571 and its true value is x = 3.1415926. Find the absolute and relative errors.

Ans.: True value of $\pi(x) = 3.1415926$

Approximate value of $\pi(x_1) = 3.1428571$

Absolute error is given by –

$$E_a = |x - x_1|$$

$$= |3.1415926 - 3.1428571|$$

$$= 0.0012645$$

Relative error is given by –

$$E_r = \left|\frac{x - x_1}{x}\right|$$

$$= \left|\frac{3.1415926 - 3.1428571}{3.1415926}\right|$$

$$= \left|\frac{0.0012645}{3.1415926}\right|$$

$$= 0.0004025$$

Q.2. Let x = 0.00458529 find the absolute error if x is truncated to three decimal digits.

Ans.: $x = 0.00458529 = 0.458529 \times 10^{-2}$ [in normalized floating point form]

$x_1 = 0.458 \times 10^{-2}$ [after truncating to three decimal places]

Absolute error $= |x - x_1|$

$$= |0.458529 \times 10^{-2} - 0.458 \times 10^{-2}|$$

$$= 0.000529 \times 10^{-2}$$

$$= 0.000529\ E - 2$$

$$= 0.529\ E - 5$$

Q.3. Let the solution of a problem be $x_a = 35.25$ with relative error in the solution atmost 2% find the range of values upto 4 decimal digits, within which the exact value of the solution must lie.

Ans.: We are given that the approximate solution of the problem is $(x_a) = 35.25$ and it has relative error upto 2% so

$$\left|\frac{x-35.25}{x}\right| < 0.02$$

$$= -0.02 < \frac{x-35.25}{x} < 0.02$$

Case-I : if $-0.02x < \frac{x-35.25}{x}$

$$\Rightarrow -0.02x < x - 35.25$$

$$\Rightarrow 35.25 < x + 0.02x$$

$$\Rightarrow 35.25 < x\,(1 + 0.02)$$

$$\Rightarrow 35.25 < x\,(1.02)$$

$$\Rightarrow 35.25 < 1.02x$$

$$\Rightarrow \frac{35.25}{1.02} < x$$

$$\Rightarrow x > 34.5588 \qquad \text{\_\_\_ (1)}$$

Case-II: if $\frac{x-35.25}{x} < 0.02$

$$\Rightarrow x - 35.25 < 0.02x$$

$$\Rightarrow x - 0.02x < 35.25$$

$$\Rightarrow 0.98x < 35.25$$

$$\Rightarrow x < \frac{35.25}{0.98}$$

$$\Rightarrow x < 35.9693 \qquad \text{\_\_\_ (2)}$$

From equation (1) and (2) we have $34.5588 < x < 35.9693$

∴ The required range is (34.5588, 35.9693)

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Chapter-2

Bisection Method

Q.1. Find real root of the equation x^3 - 5x + 3 upto three decimal digits.

Ans.: Here $f(x) = x^3 - 5x + 3$

$f(0) = 0 - 0 + 3 = 3\ f(x_0)$ (say)

$f(1) = 1 - 5 + 3 = -1 = f(x_1)$ (say)

Since $f(x_0), f(x_1) < 0$ so the root of the given equation lies between 0 and 1

So, $x_2 = \frac{x_0 + x_1}{2} = \frac{0+1}{2} = 0.5$

Now, $f(x_2) = f(0.5)$

$= (0.5)^3 - 5\ (0.5) + 3$

$= 0.125 - 2.5 + 3$

$= 0.625$ (which is positive)

$\because$ $f(x_1).f(x_2) < 0$

So, $x_3 = \frac{x_1 + x_2}{2} = \frac{1 + 0.5}{2} = 0.75$

Now, $f(x_3) = f(0.75)$

$= (0.75)^3 - 5\ (0.75) + 3$

$= 0.4218 - 3.75 + 3$

$= -\ 0.328$ (which is negative)

$\because$ $f(x_2).f(x_3) < 0$

So, $x_4 = \frac{x_2 + x_3}{2} = \frac{0.5+0.75}{2} = 0.625$

Now, $f(x_4) = f(0.625)$

$= (0.625)^3 - 5\ (0.625) + 3$

$= 0.244 - 3.125 + 3$

$= 0.119$ (which is positive)

$\because \quad f(x_3).f(x_4) < 0$

So, $x_5 = \dfrac{x_3 + x_4}{2} = \dfrac{0.75 + 0.625}{2} = 0.687$

Now, $f(x_5) = f(0.687)$

$= (0.687)^3 - 5\,(0.687) + 3$

$= -\,0.1108$ (which is negative)

$\because \quad f(x_4).f(x_5) < 0$

So, $x_6 = \dfrac{x_4 + x_5}{2} = \dfrac{0.625 + 0.687}{2} = 0.656$

Now, $f(x_6) = f(0.656)$

$= (0.656)^3 - 5\,(0.656) + 3$

$= 0.0023$ (which is positive)

$\because \quad f(x_5).f(x_6) < 0$

So, $x_7 = \dfrac{x_5 + x_6}{2} = \dfrac{0.687 + 0.656}{2} = 0.671$

Now, $f(x_7) = f(0.671)$

$= (0.671)^3 - 5\,(0.671) + 3$

$= -\,0.0528$ (which is negative)

$\because \quad f(x_6).f(x_7) < 0$

So, $x_8 = \dfrac{x_6 + x_7}{2} = \dfrac{0.656 + 0.671}{2} = 0.663$

Now, $f(x_8) = f(0.663)$

$= (0.663)^3 - 5\,(0.663) + 3$

$= 0.2920 - 3.315 + 3$

$= -\,0.023$ (which is negative)

$\because \quad f(x_6).f(x_8) < 0$

So, $x_9 = \dfrac{x_6 + x_8}{2} = \dfrac{0.656 + 0.663}{2} = 0.659$

Now, $f(x_9) = f(0.659)$

$= (0.659)^3 - 5\,(0.659) + 3$

$= -\,0.0089$ (which is negative)

$\therefore \quad f(x_6).f(x_9) < 0$

So, $\quad x_{10} = \dfrac{x_6 + x_9}{2} = \dfrac{0.656 + 0.659}{2} = 0.657$

Now, $f(x_{10}) = f(0.657)$

$= (0.657)^3 - 5\,(0.657) + 3$

$= -\,0.00140$ (which is negative)

$\because \quad f(x_6).f(x_{10}) < 0$

So, $\quad x_{11} = \dfrac{x_6 + x_{10}}{2} = \dfrac{0.656 + 0.657}{2} = 0.656$

Now, $f(x_{11}) = f(0.656)$

$= (0.656)^3 - 5\,(0.656) + 3$

$= 0.2823 - 3.28 + 3$

$= 0.00230$ (which is positive)

$\because \quad f(x_{11}).f(x_{10}) < 0$

So, $\quad x_{12} = \dfrac{x_{10} + x_{11}}{2} = \dfrac{0.657 + 0.656}{2} = 0.656$

Since x_{11} and x_{12} both same value. Therefore if we continue this process we will get same value of x so the value of x is 0.565 which is required result.

Q.2. Find real root of the equation cos x - xe^x = 0 correct upto four decimal places.

Ans.: Since, $f(x) = \cos x - xe^x$

So, $\quad f(0) = \cos 0 - 0e^0 = 1$ (which is positive)

And $\quad f(1) = \cos 1 - 1e^1 = -2.1779$ (which is negative)

$\because \quad f(0).f(1) < 0$

Hence the root of are given equation lies between 0 and 1.

let $f(0) = f(x_0)$ and $f(1) = f(x_1)$

So, $\quad x_2 = \dfrac{x_0 + x_1}{2} = \dfrac{0 + 1}{2} = 0.5$

Now, $f(x_2) = f(0.5)$

$f(0.5) = \cos(0.5) - (0.5)e^{0.5}$

$= 0.05322$ (which is positive)

$\because \quad f(x_1).f(x_2) < 0$

So, $x_3 = \frac{x_1 + x_2}{2} = \frac{1 + 0.5}{2} = \frac{1.5}{2} = 0.75$

Now, $f(x_3) = f(0.75)$

$= \cos(0.75) - (0.75)e^{0.75}$

$= -0.856$ (which is negative)

$\because \quad f(x_2).f(x_3) < 0$

So, $x_4 = \frac{x_2 + x_3}{2} = \frac{0.5 + 0.75}{2} = 0.625$

$f(x_4) = f(0.625)$

$= \cos(0.625) - (0.625)e^{(0.625)}$

$= -0.356$ (which is negative)

$\because \quad f(x_2).f(x_4) < 0$

So, $x_5 = \frac{x_2 + x_4}{2} = \frac{0.5 + 0.625}{2} = 0.5625$

Now, $f(x_3) = f(0.5625)$

$= \cos(0.5625) - 0.5625e^{0.5625}$

$= -0.14129$ (which is negative)

$\because \quad f(x_2).f(x_5) < 0$

So, $x_6 = \frac{x_2 + x_5}{2} = \frac{0.5 + 0.5625}{2} = 0.5312$

Now, $f(x_6) = f(0.5312)$

$= \cos(0.5312) - (0.5312)e^{0.5312}$

$= -0.0415$ (which is negative)

$\because \quad f(x_2).f(x_6) < 0$

So, $x_7 = \frac{x_2 + x_6}{2} = \frac{0.5 + 0.5312}{2} = 0.5156$

Now, $f(x_7) = f(0.5156)$

$= \cos(0.5156) - (0.5156)e^{0.5156}$

$= 0.006551$ (which is positive)

$\because \quad f(x_6).f(x_7) < 0$

So, $x_8 = \frac{x_6 + x_7}{2} = \frac{0.513 + 0.515}{2} = 0.523$

Now, $f(x_8) = f(0.523)$

$= \cos(0.523) - (0.523)e^{0.523}$

$= -0.01724$ (which is negative)

$\because \quad f(x_7).f(x_8) < 0$

So, $(x_9) = \dfrac{x_7 + x_8}{2} = \dfrac{0.515 + 0.523}{2} = 0.519$

Now, $f(x_9) = f(0.519)$

$= \cos(0.519) - (0.519)e^{0.519}$

$= -0.00531$ (which is negative)

$\because \quad f(x_7).f(x_9) < 0$

So, $(x_{10}) = \dfrac{x_7 + x_9}{2} = \dfrac{0.515 + 0.519}{2} = 0.5175$

Now, $f(x_{10}) = f(0.5175)$

$= \cos(0.5175) - (0.5175)e^{0.5175}$

$= 0.0006307$ (which is positive)

$\because \quad f(x_9).f(x_{10}) < 0$

So, $x_{11} = \dfrac{x_9 + x_{10}}{2} = \dfrac{0.5195 + 0.5175}{2} = 0.5185$

Now, $f(x_{11}) = f(0.5185)$

$= \cos(0.5185) - (0.5185)e^{0.5185}$

$= -0.002260$ (which is negative)

$\because \quad f(x_{10}).f(x_{11}) < 0$

So, $x_{12} = \dfrac{x_{10} + x_{11}}{2} = \dfrac{0.5175 + 0.5185}{2} = 0.5180$

Hence the root of the given equation upto 3 decimal places is x = 0.518

Thus the root of the given equation is x = 0.518

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Chapter-3

Regula Falsi Method

Q.1. Find the real root of the equation $x \log_{10} x - 1.2 = 0$ correct upto four decimal places.

Ans.: Given $f(x) = x \log_{10} x - 1.2$ _ _ _ (1)

In this method following formula is used -

$$x_{n+1} = x_n - \frac{(x_n - x_{n-1})\ f(x_n)}{(f(x_n) - f(x_{n-1}))} \qquad \text{\_ \_ \_ (2)}$$

Taking x = 1 in eq.(1)

$f(1) = 1.\ \log_{10} 1 - 1.2$

$= -2$ (which is negative)

Taking x = 2 in eq.(1)

$f(2) = 2.\ \log_{10} 2 - 1.2$

$= -0.5979$ (which is negative)

Taking x = 3 in eq.(1)

$f(3) = 3.\ \log_{10} 3 - 1.2$

$= 0.2313$ (which is positive)

$\because \quad f(2).f(3) < 0$

So the root of the given equation lies between 2 and 3.

let $x_1 = 2$ and $x_2 = 3$

$\therefore \quad f(x_1) = f(2) = -0.5979$

And $\quad f(x_2) = f(3) = 0.2313$

Now we want to find x_3 so using eq.(2)

$$x_3 = x_2 - \frac{(x_2 - x_1)\ f(x_2)}{f(x_2) - f(x_1)}$$

$$= 3 - \frac{(3-2)\ \times\ (0.2313)}{0.2313 - (-0.5979)}$$

$$= 3 - \frac{0.2313}{0.8292}$$

$$= 3 - 0.2789 = 2.7211$$

$$f(x_3) = f(2.7211)$$

$$= 2.7211 \log_{10} 2.7211 - 1.2$$

$$= -0.01701 \text{ (which is negative)}$$

$\because \quad f(x_2).f(x_3) < 0$

Now to find x_4 using equation (2)

$$x_4 = x_3 - \frac{(x_3 - x_2)\ f(x_3)}{f(x_3) - f(x_2)}$$

$$= 2.7211 - \frac{(2.7211 - 3) \times (-0.0170)}{(-0.0170 - 0.2313)}$$

$$= 2.7211 - \frac{0.004743}{0.2483}$$

$$= 2.7211 + 0.01910 = 2.7402$$

Now

$$f(x_4) = f(2.7402)$$

$$= 2.7402 \log_{10} 2.7402 - 1.2$$

$$= -0.0003890 \text{ (which is negative)}$$

$\because \quad f(x_2).f(x_4) < 0$

Now to find x_5 using equation (2)

$$x_5 = x_4 - \frac{(x_4 - x_2)\ f(x_4)}{[f(x_4) - f(x_2)]}$$

$$= 2.7402 - \frac{(2.7402 - 3)}{(-0.0004762 - 0.2313)} \times (-0.0004762)$$

$$= 2.7402 + \frac{(-0.2598)(-0.0004762)}{0.2317}$$

$$= 2.7402 + \frac{(0.0001237)}{0.2317}$$

$$= 2.7402 + 0.0005341 = 2.7406$$

$$f(x_5) = f(2.7406)$$

$= 2.7406 \log_{10} 2.7406 - 1.2$

$= -0.0000402$ (which is negative)

$\because \quad f(x_2).f(x_5) < 0$

To find x_6 using equation (2)

$$x_6 = x_5 - \frac{(x_5 - x_2)\ f(x_5)}{f(x_5) - f(x_2)}$$

$$= 2.7406 + \frac{(2.7406 - 3)\ \times\ (-0.000040)}{(-0.00004) - (0.2313)}$$

$$= 2.7406 + 0.000010 = 2.7406$$

$\therefore$ The approximate root of the given equation is 2.7406 which is correct upto four decimals.

Q.2. Find the real root of the equation $x^3 - 2x - 5 = 0$ correct upto four decimal places.

Ans.: Given equation is

$$f(x) = x^3 - 2x - 5 \qquad \text{\_ \_ \_ (1)}$$

In this method following formula is used :-

$$x_{n+1} = x_n - \frac{(x_n - x_{n-1})\ f(x_n)}{[f(x_n) - f(x_{n-1})]} \qquad \text{\_ \_ \_ (2)}$$

Taking x = 1 in equation (1)

$f(1) = 1 - 2 - 5 = -6$ (which is negative)

Taking x = 2 in equation (1)

$f(2) = 8 - 4 - 5 = -1$ (which is negative)

Taking x = 3

$f(3) = 27 - 6 - 5 = 16$ (which is positive)

Since $f(2).f(3) < 0$

So the root of the given equation lies between 2 and 3.

Let $x_1 = 2$ and $x_2 = 3$

$f(x_1) = f(2) = -1$

and $f(x_2) = f(3) = 16$

Now to find x_3 using equation (2)

$$x_3 = x_2 - \frac{(x_2 - x_1)}{[f(x_2) - f(x_1)]} f(x_2)$$

$$= 3 - \frac{(3 - 2)}{16 + 1} \times 16$$

$$= 3 - \frac{16}{17} = 2.0588$$

$$f(x_3) = (2.0558)^3 - 2\,(2.0588) - 5$$

$$= 8.7265 - 4.1176 - 5$$

$$= -0.3911 \text{ (which is negative)}$$

$\therefore \quad f(x_2).f(x_3) < 0$

Now to find x_4 using equation (2)

$$x_4 = x_3 - \frac{(x_3 - x_2)}{[f(x_3) - f(x_2)]} \times f(x_3)$$

$$= 2.0588 - \frac{(2.0588 - 3)}{-0.3911 - 16} \times (-0.3911)$$

$$= 2.0588 + \frac{(-0.9412)\times(-0.3911)}{16.3911} = 2.0812$$

$\therefore \quad f(x_4) = 9.0144 - 4.1624 - 5$

$$= -0.148 \text{ (which is negative)}$$

So $\quad f(x_2) \,.\, f(x_4) < 0$

Now using equation (2) to find x_5

$$x_5 = x_4 - \frac{(x_4 - x_2)}{[f(x_4) - f(x_2)]} \times f(x_4)$$

$$= 2.0812 - \frac{(2.0812 - 3)}{(-0.148 - 16)} \times (-0.148)$$

$$= 2.0812 + \frac{(-0.9188)\times(-0.148)}{16.148}$$

$$= 2.0812 + 8.4210 \text{ x } \frac{(x_5 - x_2)\times f(x_5)}{[f(x_5) - f(x_2)]} 10^{-3}$$

$$= 2.0896$$

. $f(x_5) = 9.1240 - 4.1792 - 5$

$= -0.0552$ (which is negative)

$f(x_2).f(x_5) < 0$

Now using equation (2) to find x_6

$$x_6 = x_5 - \frac{(x_5 - x_2) \times f(x_5)}{f(x_5) - f(x_2)}$$

$$= 2.0896 - \frac{(2.0896 - 3)}{(-0.0552 - 16)} \times (-0.0552)$$

$$= 2.0896 + \frac{(0.05025)}{16.0552}$$

$$= 2.0927$$

$\therefore$ $f(x_6) = 9.1647 - 4.1854 - 5$

$= -0.0207$ (which is negative)

So $f(x_2).f(x_6) < 0$

Now using equation (2) to find x_7

$$x_7 = x_6 - \frac{(x_6 - x_2)}{f(x_6) - f(x_2)} \times f(x_6)$$

$$= 2.0927 - \frac{(2.0927 - 3)}{(-0.0207 - 16)} \times (-0.0207)$$

$$= 2.0927 + \frac{(-0.9073)\ (-0.0207)}{16.0207}$$

$$= 2.0927 + 1.1722 \times 10^{-3}$$

$$= 2.0938$$

Now $f(x_7) = 9.1792 - 4.1876 - 5$

$= -0.0084$ (which is negative)

So $f(x_2).f(x_7) < 0$

Now using equation (2) to find x_8

$$x_8 = x_7 - \frac{(x_7 - x_2)}{f(x_7) - f(x_2)} \times f(x_7)$$

$$= 2.0938 - \frac{(2.0938 - 3)}{(-0.0084 - 16)} \times (-0.0084)$$

$$= 2.0938 + \frac{(-0.9062)(-0.0084)}{16.0084}$$

$$= 2.0938 + 4.755 \times 10^{-4}$$

$$= 2.09427$$

$\therefore \quad f(x_8) = 9.1853 - 4.18854 - 5$

$= -0.00324$ (which is negative)

So $\quad f(x_2).f(x_8) < 0$

Now using equation (2) to find x_9

$$x_9 = x_8 - \frac{(x_8 - x_2)}{f(x_8) - f(x_2)} \times f(x_8)$$

$$= 2.09427 - \frac{(2.09427 - 3)}{(-0.00324 - 16)} \times (-0.00324)$$

$$= 2.09427 - \frac{(-0.90573)(-0.00324)}{16.00324}$$

$$= 2.0944$$

$\therefore$ The real root of the given equation is 2.094 which is correct upto three decimals.

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Chapter-4

Secant Method

Note : In this method following formula is used to find root –

$$x_{n+1} = x_n - \frac{(x_n - x_{n-1}) f(x_n)}{[f(x_n) - f(x_{n-1})]} \qquad \text{\_ \_ \_ (1)}$$

Q.1. Find the root of the equation $x^3 - 5x^2 - 17x + 20$ [use Secant Method] correct upto four decimals.

Ans.: Given $f(x) = x^3 - 5x^2 - 17x + 20$ _ _ _ (2)

Taking x = 0 in equation (1)

$f(0) = 20$

Now taking x = 1

$f(1) = 1 - 5 - 17 + 20$

$= -1$

Since $f(0) = 20$ (positive) and $f(1) = -1$ (which is negative) so the root of the given equation lies between 0 and 1.

Let $x_1 = 0$ and $x_2 = 1$

$\therefore \quad f(x_1) = 20$ and $f(x_2) = -1$

using equation (1) to find x_3

$$x_3 = x_2 - \frac{(x_2 - x_1)}{f(x_2) - f(x_1)} \times f(x_2)$$

$$= 1 - \frac{(1 - 0)}{(-1) - 20} \times (-1)$$

$$= 1 + \frac{(1)}{(-21)} = 1 - \frac{1}{21}$$

$$= 0.9523$$

$\therefore \quad f(x_3) = f(0.9523)$

$= (0.9523)^3 - 5\,(0.9523)^2 - 17\,(0.9523) + 20$

$= 0.8636 - 4.5343 - 16.1891 + 20$

$= 0.1402$ (which is positive)

Using equation (1) to find x_4

$$x_4 = x_3 - \frac{(x_3 - x_2)}{f(x_3) - f(x_2)} \times f(x_3)$$

$$= 0.9523 - \frac{(0.9523 - 1)}{[0.1402 - (-1)]} \times 0.1402$$

$$= 0.9523 - \frac{(-0.0477)\,(0.1402)}{(1.1402)}$$

$$= 0.9523 + 0.005865 \qquad = 0.9581$$

$f(x_4) = (0.9581)^3 - 5\,(0.9581)^2 - 17\,(0.9581) + 20$

$= 0.8794 - 4.5897 - 16.2877 + 20$

$= 0.0020$ (which is positive)

$$x_5 = x_4 - \frac{(x_4 - x_3)}{f(x_4) - f(x_3)} \times f(x_4)$$

$$= 0.9581 - \frac{(0.9581 - 0.9523)}{(0.0020) - (0.1402)} \times 0.0020$$

$$= 0.9581$$

Hence the root of the given equation is 0.9581 which is correct upto four decimal.

Q.2. Given that one of the root of the non-linear equation $\cos x - xe^x = 0$ lies between 0.5 and 1.0 find the root correct upto three decimal places, by Secant Method.

Ans.: Given equation is $f(x) = \cos x - xe^x$

And $x_1 = 0.5$ and $x_2 = 1.0$

$f(x_1) = \cos(0.5) - (0.5)\, e^{0.5}$

$= 0.87758 - 0.82436$

$= 0.05321$

Now $f(x_2) = \cos(1) - (1)\, e^1$

$= 0.54030 - 2.71828$

$= -2.1780$

Now to calculate x_3 using equation (1)

$$x_3 = x_2 - \frac{(x_2 - x_1)}{f(x_2) - f(x_1)} \times f(x_2)$$

$$= 1 - \frac{(1 - 0.5)}{(-2.1780 - 0.05321)} \times (-2.1780)$$

$$= 1 - \frac{(0.5)\,(2.1780)}{2.23121}$$

$$= 1 - 0.48807$$

$$= +0.51192$$

$\therefore \quad f(x_3) = f(0.51192)$

$$= \cos(0.51192) - (0.51192)e^{0.51192}$$

$$= 0.87150 - 0.85413$$

$$= 0.01767$$

Now for calculating x_4 using equation (1)

$$x_4 = x_3 - \frac{(x_3 - x_2)}{f(x_3) - f(x_2)} \times f(x_3)$$

$$= 0.51192 - \frac{(0.51192 - 1)}{(0.1767) - (-2.1780)} \times 0.01767$$

$$= 0.51192 - \frac{(-0.48808)\,(0.01767)}{2.19567}$$

$$= 0.51192 + \frac{0.0086243}{2.19567}$$

$$= 0.51192 + 0.003927$$

$$= 0.51584$$

$\therefore \quad f(x_4) = \cos(0.51584) - (0.51584)e^{0.51584}$

$$= 0.86987 - 0.86405$$

$$= 0.005814 \text{ (which is positive)}$$

Now for calculating x_5 using equation (1)

$$x_5 = x_4 - \frac{(x_4 - x_3)}{f(x_4) - f(x_3)} \times f(x_4)$$

$$= 0.51584 - \frac{(0.51584 - 0.51192)}{(0.005814 - 0.01767)} \times 0.005814$$

$$= 0.51584 - \frac{0.00392}{(-0.01185)} \times (0.005814)$$

$= 0.51584 + 0.001923$

$= 0.51776$

$= 0.5178$

Now $f(x_5) = \cos(0.5178) - (0.5178)e^{0.5178}$

$= 0.8689 - 0.8690$

$= -0.00001$

$= -0.0000$ (upto four decimals)

Hence the root of the given equation is x = 0.5178 (which is correct upto four decimal places)

This process cannot be proceed further because $f(x_5)$ vanishes.

□ □ □

Chapter-5

Newton Raphson Method

Hint : Formula uses in this method is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Q.1. Find the root of the equation $x^2 - 5x + 2 = 0$ correct upto 5 decimal places. (use Newton Raphson Method.)

Ans.: : Given $f(x) = x^2 - 5x + 2 = 0$

Taking x = 0

$f(0) = 2$ (which is positive)

Taking x = 1

$f(1) = 1 - 5 + 2 = -2$ (which is negative)

$f(0) . f(1) < 0$

∴ The root of the given equation lies between 0 and 1

Taking initial approximation as

$$x_1 = \frac{0+1}{2} = 0.5$$

$f(x) = x^2 - 5x + 2$

$f'(x) = 2x - 5$

Since $x_1 = 0.5$

$f(x_1) = (0.5)^2 - 5(0.5) + 2$

$= 0.25 - 2.5 + 2$

$= -0.25$

$f'(x_1) = 2(0.5) - 5$

$= 1 - 5$

$= -4$

Now finding x_2

$$x_2 = 0.5 - \frac{(-0.25)}{-4}$$

$$= 0.5 - \frac{0.25}{4}$$

$$= 0.4375$$

$$f(x_2) = (0.4375)^2 - 5(0.4375) + 2$$

$$= 0.19140 - 2.1875 + 2$$

$$= 0.003906$$

$$f'(x_2) = 2(0.4375) - 5$$

$$= -4.125$$

Now finding x_3

$$x_3 = x_2 - \frac{f(x_2)}{f^1(x_2)}$$

$$= 0.4375 - \frac{0.003906}{(-4.125)}$$

$$= 0.4375 + 0.0009469$$

$$= 0.43844$$

$$f(x_3) = (0.43844)^2 - 5(0.43844) + 2$$

$$= 0.19222 - 2.1922 + 2$$

$$= 0.00002$$

$$f'(x_3) = 2 \times (0.43844) - 5$$

$$= -4.12312$$

$$x_4 = x_3 - \frac{f(x_3)}{f^1(x_3)}$$

$$= 0.43844 - \frac{0.00002}{(-4.12312)}$$

$$= 0.43844 + 0.00000485$$

$$= 0.43844$$

Hence the root of the given equation is 0.43844 which is correct upto five decimal places.

Q.2. Apply Newton Raphson Method to find the root of the equation 3x – cos x – 1 = 0 correct the result upto five decimal places.

Ans.: Given equation is

$$f(x) = 3x - \cos x - 1$$

Taking x = 0

$$f(0) = 3(0) - \cos 0 - 1$$
$$= -2$$

Now taking x = 1

$$f(1) = 3(1) - \cos(1) - 1$$
$$= 3 - 0.5403 - 1$$
$$= 1.4597$$

Taking initial approximation as

$$x_1 = \frac{0+1}{2} = 0.5$$

$$f(x) = 3x - \cos x - 1$$
$$f'(x) = 3 + \sin x$$

At $x_1 = 0.5$

$$f(x_1) = 3(0.5) - \cos(0.5) - 1$$
$$= 1.5 - 0.8775 - 1$$
$$= -0.37758$$
$$f'(x_1) = 3 - \sin(0.5)$$
$$= 3.47942$$

Now to find x_2 using following formula

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$
$$= 0.5 - \frac{(-0.37758)}{(3.47942)}$$
$$= 0.5 + 0.10851$$
$$= 0.60852$$

$$f(x_2) = 3(0.60852) - \cos(0.60852) - 1$$
$$= 1.82556 - 0.820494 - 1$$
$$= 0.005066$$
$$f'(x_2) = 3 + \sin(0.60852)$$

$= 3.57165$

Now finding x_3

$$x_3 = 0.60852 - \frac{(0.005066)}{(3.57165)}$$

$= 0.60852 - 0.0014183$

$= 0.60710$

$f(x_3) = 3\,(0.60710) - \cos\,(0.60710) - 1$

$= 1.8213 - 0.821305884 - 1$

$= -\,0.00000588$

$f'\,(x_3) = 3 + \sin\,(0.60710)$

$= 3 + 0.57048$

$= 3.5704$

Now to find x_4 using following formula

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 0.60710 - \frac{(-0.00000588)}{3.5704}$$

$= 0.60710 + 0.00000164$

$= 0.60710$

Which is same as x_3

Hence the root of the given equation is $x = 0.60710$ which is correct upto five decimal places.

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